Q- A roller coaster car has a mass of 500kg when fully loaded with passengers. The radius of the coaster is 10m.
(a) If the vehicle has a speed of 20.0 m/s at the lowest point, what is the force exerted by the track on the car at this point?

A roller coaster is a vertical circular track on which cars can move almost without friction. When a body moves on a vertical circular track without friction, the forces acting on it are (1) its weight mg vertically downward and (2) the normal force of the track N towards the center of the track. As to move on a circular path a centripetal force is required, the resultant of the component of the weight towards center and the normal force will provide this force.

When the body is at the lowest point of the track the normal force N is upward and the weight mg of the body is acting downwards hence their resultant will be (N - mg) upward and is providing necessary centripetal force.

Let the radius of the coaster be R, mass of the car is m and at the lowest point of the track its velocity is V_L. The required centripetal force is given by the resultant (N - mg) and hence we have

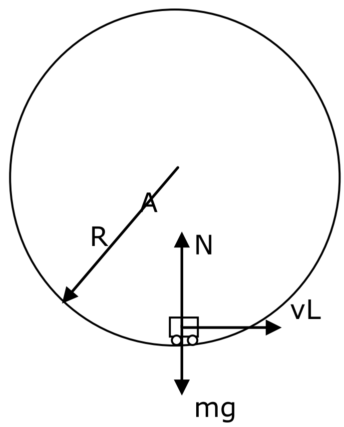
$$(N - mg) = mv_L^2/R$$

Gives $N = mg + (mv_L^2/R)$

Substituting the values we have

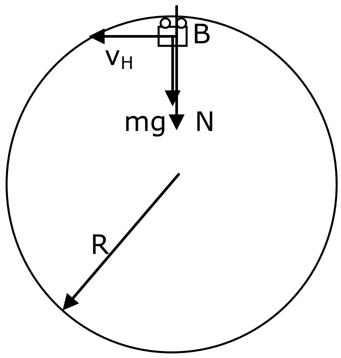
$$N = 500*9.8 + 500*(20.0)^2/10.0$$

Or $N = 4900 + 500*400/10 = 4900 + 20000 = 24900$ N



Hence the force exerted by the track on the car will be $2.49*10^4$ N (upward)

(b) What is the maximum speed the vehicle can have at point B and still remain on the track?



At the highest point B of the track the velocity of the car is v_h. At this position the normal force and the weight of the car both are in downward direction and hence their resultant (N + mg) will provide necessary centripetal force. Hence we have

$$N + mg = mv_H^2/R$$

Here mg is a constant and hence as the velocity of the car at point B will be less normal force will be less. At some particular value of velocity v_H the normal force N will become zero. In this situation the weight of the car mg only will provide the necessary centripetal force. If the velocity will be further less the contact between the track and the car will be lost before reaching this point, thus the minimum speed at which the car can complete the loop without losing contact with the track will be such that N is just zero at B. Thus for minimum speed at the highest point

$$0 + mg = mv_H^2/R$$

Or $v_H = \sqrt{gR}$

Or $v_H = \sqrt{9.8 * 10} = 9.9\ m/s$