Q- A 100 μF capacitor is connected in turn to two different sinusoidal voltages of the same amplitude V = 1.0V but different frequencies with f_1 = 50Hz and f_2 = 5 kHz.

An alternating current can pass through capacitors and inductors. As a conductor offers oppose to the current through it and this oppose is measured by its resistance, in the same way in alternating current circuits the capacitors and inductors too offers oppose to the current. As there is no energy loss against these oppose and the current and voltage across these are not in same phase as resistors, these oppose are called reactance. The reactance may be capacitive and inductive.

The reactance depends on the frequency of alternating current (or voltage) and the capacitive reactance is given by

$$X_C = \frac{1}{C\omega}$$

Where C is the capacitance and ω is the angular frequency.

(a) Calculate the reactance X_1 and X_2.

The angular frequency w is related to the frequency by $\omega = 2\pi f$ hence the capacitive reactance will be

$$X_C = \frac{1}{C\omega} = \frac{1}{2\pi f * C}$$

Gives $X_1 = \frac{1}{2\pi f_1 * C} = \frac{1}{2*3.14*50*100*10^{-6}} = 31.85\Omega$

And $X_2 = \frac{1}{2\pi f_2 * C} = \frac{1}{2*3.14*5000*100*10^{-6}} = 0.3185\Omega$

Clearly with increase in frequency the capacitive reactance decreases.

(b) By what factor is X_1 greater than X_2 ?

The ratio X_1/X_2 will be same as the ratio of f_2/f_1 and hence it is 100.

(c) Calculate the amplitudes of the associated currents, I_1 and I_2.

The amplitude of the current is given by according to Ohm's law as

I = V/X

Hence

$$I_1 = \frac{V}{X_1} = \frac{1.0}{31.85} = 0.0314 \text{ A} = 31.4 \text{ mA}.$$

And $I_2 = \frac{V}{X_2} = \frac{1.0}{0.3185} = 3.14\,\text{A}$