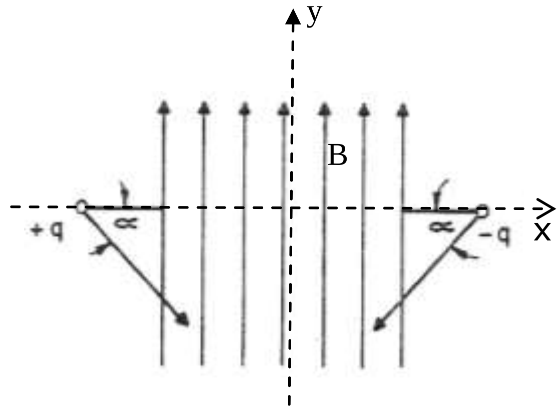
Q- Positively and negatively charged particles are simultaneously “shot” with velocity v through the magnetic field B as in figure in the plane of the paper. Find the magnitude and direction of the force acting on each particle.

The force acting on an electric charge moving in a magnetic field is called Laurent’s force and is given by

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right)$$



Considering x and y directions as in figure and the z direction out of the page (right handed system), the velocity and the magnetic field can be written in the components form as

a) For the positive charge

$$\vec{v} = v\cos\alpha.\hat{i} - v\sin\alpha.\hat{j}$$

As the x component of velocity is in positive x direction and y component is in negative y direction.

And $\vec{B} = B.\hat{j}$

Where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors in x, y and z directions respectively.

Writing the force as the vector product we have

$$\vec{F}_+ = q\left(\vec{v} \times \vec{B}\right) = q\left[v\left(\cos\alpha.\hat{i} - \sin\alpha.\hat{j}\right) \times B.\hat{j}\right]$$

Or $\vec{F}_+ = qvB\left[\left(\cos\alpha.\hat{i} - \sin\alpha.\hat{j}\right) \times .\hat{j}\right]$

Or $\vec{F}_+ = qvB\left[\cos\alpha.\left(\hat{i} \times .\hat{j}\right) - \sin\alpha.\left(\hat{j} \times .\hat{j}\right)\right]$

Or $\vec{F}_+ = qvB\left[\cos\alpha.\hat{k} - \sin\alpha.\vec{0}\right]$ [as $\hat{i}$ X $\hat{j} = \hat{k}$ and $\hat{j}$ X $\hat{j}$ = 0]

Or $\vec{F}_+ = qvB\cos\alpha.\hat{k}$

Hence the force on the + q charge is in positive z direction or out of the paper.

b) For the negative charge

$$\vec{v} = -v\cos\alpha.\hat{i} - v\sin\alpha.\hat{j}$$

As the x component of velocity is in negative x direction and y component is also in negative y direction.

And $\vec{B} = B.\hat{j}$

Writing the force for -q charge we have

$$\vec{F}_{-} = -q\left(\vec{v}\times\vec{B}\right) = -q\left[v\left(-\cos\alpha.\hat{i} - \sin\alpha.\hat{j}\right)\times B.\hat{j}\right]$$

Or $$\vec{F}_{-} = qvB\left[\left(\cos\alpha.\hat{i} + \sin\alpha.\hat{j}\right)\times.\hat{j}\right]$$

Or $$\vec{F}_{+} = qvB\left[\cos\alpha.\left(\hat{i}\times.\hat{j}\right) + \sin\alpha.\left(\hat{j}\times.\hat{j}\right)\right]$$

Or $$\vec{F}_{+} = qvB\left[\cos\alpha.\hat{k} + \sin\alpha.\vec{0}\right]$$ [as $\hat{i}$ X $\hat{j} = \hat{k}$ and $\hat{j}$ X $\hat{j}$ = 0]

Or $$\vec{F}_{+} = qvB\cos\alpha.\hat{k}$$

Hence the force on the - q charge is also in positive z direction or out of the paper.