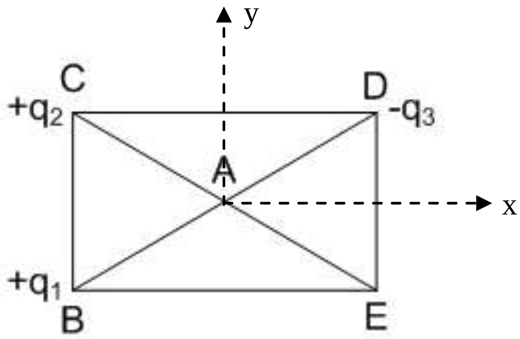
Q- Consider the rectangle below BCDE of length 24cm. and width 18cm. Position A is at the centre of the rectangle. Point charges q_1 = (+) 0.2 μC, q_2 = (+) 0.8 μC and q_3 = (-) 0.6 μC are placed at the corners B, C and D of the rectangle as shown. Find the magnitude and direction of the electric field at position A.



The strength of electric field (also called field strength of electric field) at a point is measured by the force experience per unit test charge at that point.

The field strength at a point due to a point charge Q is given by using Coulomb's law as

$$\vec{E} = \frac{Q}{4\pi \in_0 r^2}\hat{r}$$

Where r is the distance of the point from the point charge and $\hat{r}$ is the unit vector in the direction of line joining from Q to the point. Multiplying r in numerator and denominator we can write this as

$$\vec{E} = \frac{Q}{4\pi \in_0 r^3} * \vec{r}$$

Now the diagonal of the rectangle is

$$CE = \sqrt{CB^2 + BE^2} = \sqrt{18^2 + 24^2} = 30\,\text{cm}$$

Hence r = CA + BA + DA = 15 cm = 0.15 m.

The components of this displacement r along x and y from all the three charges are 0.12 cm and 0.09 cm and hence for the three charges r is having components equal in magnitudes but differs in directions.

[$\hat{i}$ and $\hat{j}$ are unit vectors in x and y directions respectively]

Thus electric field at A due to charge q_1 at B is given by

$$\vec{E}_1 = \frac{q_1}{4\pi \in_0 r^3}\left(x\hat{i} + y\hat{j}\right) = \frac{9*10^9 * \left(+0.2*10^{-6}\right)}{0.15^3}\left(0.12\hat{i} + 0.09\hat{j}\right)$$

Or $$\vec{E}_1 = 5.33*10^5 * \left(0.12\hat{i} + 0.09\hat{j}\right) = 0.64*10^5\hat{i} + 0.48*10^5\hat{j}$$

Similarly the electric field at A due to charge q_2 at C is given by

$$\vec{E}_2 = \frac{q_2}{4\pi \in_0 r^3}\left(x\hat{i} - y\hat{j}\right) = \frac{9*10^9 * \left(+0.8*10^{-6}\right)}{0.15^3}\left(0.12\hat{i} - 0.09\hat{j}\right)$$

Or $$\vec{E}_2 = 2.13*10^6 * \left(0.12\hat{i} - 0.09\hat{j}\right) = 2.56*10^5\hat{i} - 1.92*10^5\hat{j}$$

And the electric field at A due to charge q_3 at D is given by

$$\vec{E}_3 = \frac{q_3}{4\pi \in_0 r^3}\left(x\hat{i} + y\hat{j}\right) = \frac{9*10^9 * \left(-0.6*10^{-6}\right)}{0.15^3}\left(-0.12\hat{i} - 0.09\hat{j}\right)$$

Or $\vec{E}_3 = -1.6*10^6 * \left(-0.12\hat{i} - 0.09\hat{j}\right) = 1.92*10^5\hat{i} + 1.44*10^5\hat{j}$

Now according to superposition law the resultant field at a point due to number of point charges is the vector sum of the fields due to individual charges and hence the field at A due to all three charges is given by

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

Or

$$\vec{E} = \left(0.64*10^5\hat{i} + 0.48*10^5\hat{j}\right) + \left(2.56*10^5\hat{i} - 1.92*10^5\hat{j}\right) + \left(1.92*10^5\hat{i} + 1.44*10^5\hat{j}\right)$$

Or $\vec{E} = \left[\left(0.64\hat{i} + 0.48\hat{j}\right) + \left(2.56\hat{i} - 1.92\hat{j}\right) + \left(1.92\hat{i} + 1.44\hat{j}\right)\right]*10^5$

Or $\vec{E} = \left(5.12\hat{i} + 0.0\hat{j}\right)*10^5$

This shows that the y component is zero and hence the field is in x direction or along the length of the rectangle. Its magnitude is $5.12*10^5$ N/C.

Alternative method

As q_1 and q_3 are on the opposite side of A and of the opposite sign, the fields due to both will be added up and along AD with magnitude

$$E_{1,3} = \frac{q_1}{4\pi \in_0 r^2} + \frac{q_3}{4\pi \in_0 r^2} = \frac{9*10^9}{0.15^2}\left(0.2 + 0.6\right)*10^{-6} = 3.20*10^5 \text{ N/C.}$$

And field due to q2 will be along AE and its magnitude will be

$$E_2 = \frac{q_2}{4\pi \in_0 r^2} = \frac{9*10^9}{0.15^2}*0.8*10^{-6} = 3.20*10^5 \text{ N/C}$$

Both fields are equal in magnitude and symmetric to x direction the y component of the resultant will be zero hence the resultant will be along x direction.

In the right angled triangle DBE

$$\tan\angle DBE = \frac{18}{24} = 0.75, \qquad \angle DBE = 36.87^0$$

Hence angle between the two fields will be $\angle DAE = 2*36.87^0 = 73.74^0$

Thus the magnitude of the resultant field at A will be

$$E = \sqrt{E_{1,3}^2 + E_2^2 + 2*E_{1,3}E_2\cos 73.74^0}$$

Or $E = \sqrt{\left(3.20*10^5\right)^2 + \left(3.20*10^5\right)^2 + 2*\left(3.20*10^5\right)^2\cos 73.74^0}$

Or E = $5.12*10^5$ N/C.