

Q- The G string on an instrument is 49 cm long. When played without fingering, it vibrates at a frequency of 196 Hz. The next higher notes on the C-major scale are A (220 Hz), B (247 Hz), C (262 Hz), and D (294 Hz). What distance x from the end of the string must a finger be placed to play each of these notes?

The frequency of the stretched string is given by

$$n = \frac{1}{2L} \sqrt{\frac{T}{\mu}}$$

Here L is the vibrating length of the string, T the tension in the string and μ is its mass per unit length.

For the whole length

$$196 = \frac{1}{2 * 0.49} \sqrt{\frac{T}{\mu}}$$

Gives $\sqrt{\frac{T}{\mu}} = 196 * 2 * 0.49 = 192.08 \text{ m/s}$

Let the distance vibrating length be x then

A. $220 = \frac{1}{2x_A} * 192.08$

Gives $x_A = \frac{192.08}{220 * 2} = 0.4365 \text{ m} = 43.65 \text{ cm}$

$x_A = 43.65 \text{ cm}$

B. $247 = \frac{1}{2x_B} * 192.08$

Gives $x_B = \frac{192.08}{247 * 2} = 0.3888 \text{ m} = 38.88 \text{ cm}$

$x_B = 38.88 \text{ cm}$

C. $262 = \frac{1}{2x_C} * 192.08$

Gives $x_C = \frac{192.08}{262 * 2} = 0.3665 \text{ m} = 36.65 \text{ cm}$

$x_C = 36.65 \text{ cm}$

D. $294 = \frac{1}{2x_D} * 192.08$

Gives $x_D = \frac{192.08}{294 * 2} = 0.3267 = 32.67 \text{ cm}$

$x_D = 32.67 \text{ cm}$