Q. A 170 g mass attached to a horizontal spring oscillates at a frequency of 2.0 Hz. At t = 0 s, the mass is at x = 5.0 cm and has v_x = -28 cm/s. Determine the following.

(a) the period

The period of oscillation T is the is the time required for one complete oscillation while the frequency n is the number of oscillations in one second hence the period is the inverse of frequency and given by

T = 1/n = ½ = 0.5 s

(b) the angular frequency

The angular frequency is the angular velocity of reference point which is have same period as that of the oscillator and hence is given by

$$\omega = \frac{2\pi}{T} = 2\pi * n = 2 * 3.14 * 2 = 12.56 \text{ rad/s}$$

(c) the amplitude

The amplitude is the maximum displacement from the equilibrium position. It is denoted by A. The equation of the particle executing simple harmonic motion is the relation of its displacement x as a function of time t and is given by

$x = A \sin(\omega t + \phi)$ ------------------------ (1)

Here x is the displacement from the equilibrium position, A is the amplitude, ω is the angular frequency and ϕ is the initial phase angle.

The velocity of the particle as a function of time is given by differentiating equation 1 with respect to time as

$$v_x = \frac{dx}{dt} = A\omega\cos(\omega t + \phi) \qquad \text{------------------------ (2)}$$

Now substituting the values at t = 0 in equations 1 and 2 we get

5.0 cm = A sin (12.56*0 + ϕ)

Or 5.0 cm = A sin (ϕ) ------------------------- (A)

And -28 cm/s = A*12.56 cos $(12.56 * 0 + \phi)$

Or -28 cm/s = A*12.56 cos (ϕ)

Or - 2.23 = A cos (ϕ) ------------------------- (B)

Squaring and adding the two equations we have

$$(5.0)^2 + (-2.23)^2 = A^2(\sin^2\phi + \cos^2\phi)$$

Or $A^2 = (5.0)^2 + (-2.23)^2$

Gives $A = \sqrt{(5.0)^2 + (-2.23)^2} = 5.47$ cm

(d) The phase constant

Dividing the two equations we get

Tan ϕ = 5/(-2.23) = - 2.24

Gives ϕ = 114.4 deg = 1.99 rad.

(e) The maximum speed

From equation 2 we get the velocity of the particle executing simple harmonic motion as

$$v_x = A\omega \cos(\omega t + \phi)$$

The value of the velocity will be maximum when the phase angle (ωt + ϕ) is zero and hence the maximum velocity will be

v_{max} = Aω = 5.47*12.56 = 68.7 cm/s

(f) The maximum acceleration

The acceleration of the particle as a function of time can be obtain by differentiating the equation 2 with respect to time as

$$a = \frac{dv_x}{dt} = \frac{d}{dt} A\omega \cos(\omega t + \phi) = A\omega \frac{d \cos(\omega t + \phi)}{dt} = -A\omega^2 \sin(\omega t + \phi)$$

The magnitude of the acceleration will be maximum when the value of sin$(\omega t + \phi)$ is one and hence the maximum acceleration is given by (magnitude)

a_{max} = Aω^2 = 5.47*(12.56)^2 = 862.9 cm/s^2

(g) The total energy

When the particle executes simple harmonic motion, its total energy remains constant. The potential energy stored in the system (elastic potential energy) converts in kinetic energy and visa versa. At the extreme positions (displacement A) the total energy will be potential energy and at the equilibrium position the total energy will remain as kinetic energy and hence total energy will be

$$U = \frac{1}{2} m v_{max}^2 = 0.5 * 170 * 68.7^2 = 401173.65 \text{ erg} = 0.04 \text{ J}$$

(h) the position at *t* = 0.4 s

The position of the particle is given by equation 1 as

x = A sin (ωt + ϕ)

Substituting the values from above parts the position of the particle at t = 0.4 s is given by

x = 5.47 sin (12.56*0.4 rad + 1.99 rad) = 5.47*0.669 = 3.66 cm.