

Q- A wave on a string is described by $D(x,t) = (3.0 \text{ cm}) \sin[2\pi(1 + x/(2.1 \text{ m}) - t/(0.2 \text{ s}))]$.
(a) In what direction is this wave traveling?

The equation can be written as

$$D(x,t) = (3.0 \text{ cm}) \sin \left[2\pi \left(1 + \frac{x}{2.1 \text{ m}} - \frac{t}{0.2 \text{ s}} \right) \right] \text{----- (1)}$$

Or
$$D(x,t) = (3.0 \text{ cm}) \sin \left[\left(\frac{2\pi}{2.1 \text{ m}} \right) x - \left(\frac{2\pi}{0.2 \text{ s}} \right) t + 2\pi \right]$$

Now as we know that at a moment the phase angle decrease in the direction of wave motion and here if t is constant with x the phase angle is increasing this says us that the wave is traveling in negative x direction.

(b) What are the wave speed, the frequency, and the wave number?

Comparing above equation with standard wave equation we get

$$K = \frac{2\pi}{\lambda} = \frac{2\pi}{2.1 \text{ m}} \quad \text{gives wavelength } \lambda = 2.1 \text{ m}$$

And
$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.2 \text{ s}} \quad \text{gives } T = 0.2 \text{ s}$$

Hence the wave speed

$$c = \frac{\lambda}{T} = \frac{2.1}{0.2} = 10.5 \text{ m/s}$$

Frequency

$$n = \frac{1}{T} = \frac{1}{0.2} = 5 \text{ Hz}$$

and the wave number

$$K = \frac{2\pi}{\lambda} = \frac{2\pi}{2.1 \text{ m}} = 2.992 \text{ m}^{-1}$$

(c) At $t = 0.50 \text{ s}$, what is the displacement of the string at $x = 0.20 \text{ m}$?

Substituting the values for x and t in equation (1) we get

$$D(x,t) = (3.0 \text{ cm}) \sin \left[2\pi \left(1 + \frac{0.20}{2.1 \text{ m}} - \frac{0.50}{0.2 \text{ s}} \right) \right]$$

Or
$$D(x,t) = (3.0 \text{ cm}) \sin[-2.81\pi] = (3.0 \text{ cm}) \sin[-506^\circ] = 3.0 * (-.563) = -1.69 \text{ cm}$$