Q- The potential at a point on the axis of a uniformly charged circular disk is given by

$$V = \frac{\sigma}{2\epsilon_0}\left(\sqrt{a^2 - r^2} - r\right)$$

Where σ is the surface charge density, r is the distance of the point from the center of the disk and 'a' is the radius of the disk. Find an expression for the electric field as a function of r.

In an electric field an electric charge experienced a force. If we move the charge against the field we have to apply a force equal to the force due to field and displace the charge. Thus we do work and this work remains stored in the charge in form of electrostatic potential energy. The potential energy per unit test charge is called electric potential.

Let the field at a point in the field is E then the force experienced by a charge q at that point is q*E. If now we displace the charge by distance (- dr) opposite to the field the work done against the field will be

dW = q E*(- dr)

If the increase in potential in this distance dr is dV then we get

q*dV = dW = q E*(- dr)

Gives $E = -\frac{dV}{dr}$

Hence the field strength at a point is the potential gradient at that point. The negative sign indicates that the potential decreases in the direction of electric field.

Now the potential on the axis of the disk is given by

$$V = \frac{\sigma}{2\epsilon_0}\left(\sqrt{a^2 + r^2} - r\right)$$

Thus the field at distance r is given by

$$E = -\frac{dV}{dr} = -\frac{\sigma}{2\epsilon_0}\frac{d}{dr}\left(\sqrt{a^2 + r^2} - r\right)$$

Or $E = -\frac{\sigma}{2\epsilon_0}\left(\frac{2r}{2\sqrt{a^2+r^2}} - 1\right)$

Or $E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{r}{\sqrt{a^2+r^2}}\right)$

This is the desired expression.