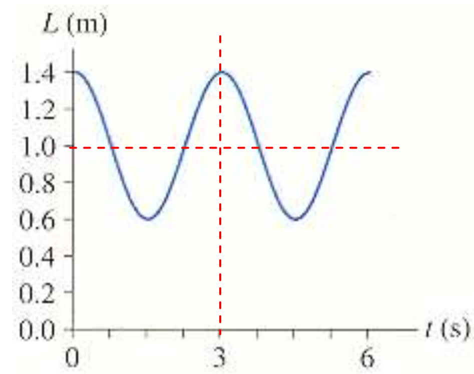
Q- Astronauts in space cannot weigh themselves by standing on a bathroom scale. Instead, they determine their mass by oscillating on a large spring. Suppose an astronaut attaches one end of a large spring to her belt and the other end to a hook on the wall of the space capsule. A fellow astronaut then pulls her away from the wall and releases her. The spring's length as a function of time is shown in Figure.

(a) What is her mass if the spring constant is 220 N/m?



As observed from the figure, the period of oscillation is 3.0 s and as we know that

$$T = 2\pi\sqrt{\frac{m}{K}}$$

Or $$m = \frac{T^2 K}{4\pi^2} = \frac{9*220}{4*3.1415^2} = 50.15\,\text{Kg}$$

(b) What is her speed when the spring's length is 1.06 m?

From the figure it is clear that the natural length of the spring is 1.0 m and the maximum elongation (Amplitude) is 0.4 m. Hence the maximum elastic potential energy or the total energy of the system is given by

$$U_T = \frac{1}{2}KA^2 = 0.5*220*0.4^2 = 17.6J$$

When the length of the spring is 1.06 m its elongation is 1.06 -1.0 = 0.06 m, hence the elastic potential energy in this position is given by

$$U = \frac{1}{2}Kx^2 = 0.5*220*0.06^2 = 0.396J$$

Thus according to law of conservation of energy

Gain in kinetic energy = loss in Potential energy

Or KE at this position = U_T - U = 17.6 - 0.396 = 17.204 J

This gives the speed v at this position as

$$\frac{1}{2}mv^2 = KE$$

Or $$v = \sqrt{\frac{2*KE}{m}} = \sqrt{\frac{2*17.204}{50.15}} = 0.828\,\text{m/s}$$