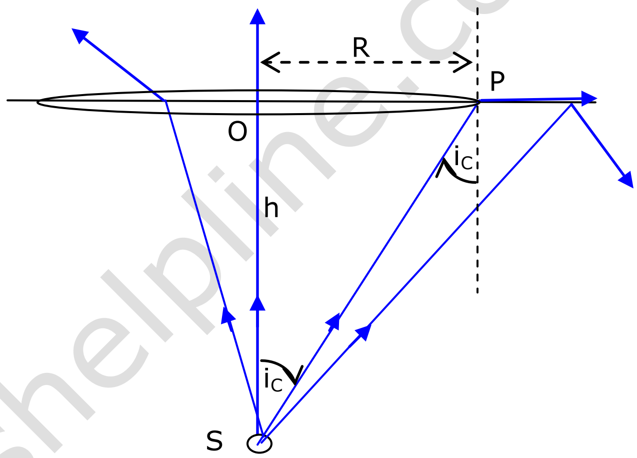
Q- A blue light point source is placed at the bottom of a 15 foot deep swimming pool, filled with a liquid of refractive index 2.3. Determine the area of the blue circular light patch seen on the surface of the liquid.

The problem is based on the phenomenon of total internal reflection. When light is going from denser to rarer medium it bents away from the normal and hence the angle of refraction is grater then angle of incidence. When angle of incident is greater than a certain value, light will totally reflected back in the same medium without getting any refracted part. This phenomenon is called total internal reflection and the maximum angle of incidence for which refraction takes is called critical angle denoted by i_C. When the angle of incidence is just near the critical the angle of refraction will be 90^0 and hence if μ is the refractive index of the denser medium relative to rarer medium we have by using Snell's law

$$\frac{1}{\mu} = \frac{\sin i_C}{\sin 90^0} \quad \text{or} \quad \sin i_C = \frac{1}{\mu}$$



Now if the point source is placed in a liquid the light coming out has to go from the liquid to the air and hence the rays incident within the distance R from the point O on the surface, just above the source, such that the angle of incidence is just equal to critical angle can only emerge out of the surface. For all other rays at distance greater then R, the angle of incidence in the liquid will be greater than critical angle and cannot be refracted and get reflected inside the liquid back. Hence light will come out only from a circular area of the surface with radius R.

As shown in the figure the measure angle PSO will be equal to the critical angle and hence we have

$$\text{Sin PSO} = \sin i_C = \frac{OP}{PS}$$

Using Pythagoras theorem for right angle triangle PSO we have

$$\sin i_C = \frac{OP}{PS} = \frac{PO}{\sqrt{PO^2 + OS^2}} = \frac{R}{\sqrt{R^2 + h^2}}$$

but as we know that $\sin i_C = \frac{1}{\mu}$ substituting in above equation we get

$$\frac{R}{\sqrt{R^2 + h^2}} = \frac{1}{\mu}$$

Or $\mu^2 R^2 = R^2 + h^2$

Or $R = \frac{h}{\sqrt{\mu^2 - 1}}$

This is the radius of the circular through which light will come out. Thus the area of this circular is given by

$$A = \pi R^2 = \frac{\pi h^2}{\mu^2 - 1} = \frac{3.1416 * 15^2}{2.3^2 - 1} = 164.77 \text{ ft}^2.$$