

Q- A patient is to be given a blood transfusion. The blood is to flow through a wide tube from a raised bottle to a needle inserted in the vein. The inside diameter of the 3.8 cm long needle is 0.4 mm and the required flow rate is 4 cm³ of blood per minute. How high should the bottle be placed above the needle? Assume the blood pressure is 20 torr above atmospheric pressure. (Density of blood = 1.05*10³ kg/m³ and its viscosity is 4*10⁻³ Pa.s)

$$1 \text{ torr} = 1 \text{ atm}/760 = 1.01 \cdot 10^5 / 760 = 132.9 \text{ Pa}$$

Or $\Delta P = 20 \text{ torr} = 2658 \text{ Pa}$

Now if the height of the bottle from the needle is h then pressure of the blood from the bottle is given by

$$P_1 = P_0 + h\rho g$$

Pressure of blood (other end of the needle) will be

$$P_2 = P_0 + P$$

Hence the pressure difference across the needle will be

$$P_1 - P_2 = P_0 + h\rho g - P_0 - P$$

Or $P_1 - P_2 = h\rho g - P$

Now the volume flow rate of a fluid through a tube of radius r and length L is given by

$$\frac{dQ}{dt} = \frac{\pi(P_1 - P_2)r^4}{8\eta L}$$

Or $\frac{dQ}{dt} = \frac{\pi(h\rho g - P)r^4}{8\eta L}$

Substituting the values in SI units we get

$$\frac{4.0 \cdot 10^{-6}}{60} = \frac{3.14 \cdot (h \cdot 1.05 \cdot 10^3 \cdot 9.8 - 2658) \cdot \left(\frac{0.4 \cdot 10^{-3}}{2}\right)^4}{8 \cdot 4 \cdot 10^{-3} \cdot 0.038}$$

Or $\frac{4.0 \cdot 10^{-6}}{60} = \frac{3.14 \cdot (10290 h - 2658) \cdot 1.6 \cdot 10^{-15}}{1.22 \cdot 10^{-3}}$

Or $(10290 h - 2658) = \frac{4.0 \cdot 10^{-6} \cdot 1.22 \cdot 10^{-3}}{60 \cdot 3.14 \cdot 1.6 \cdot 10^{-15}}$

Or $(10290 h - 2565) = 16189$

Or $h = \frac{16189 + 2658}{10290} = 1.83 \text{ m}$