Q- A hollow cylinder of radius a and length L, open at its ends carries a uniform surface charge density σ. Use the formula for electric field due to a thin uniformly charged ring; find the expression for the electric field at a point P on the axis of the cylinder at a distance b from its nearest end.

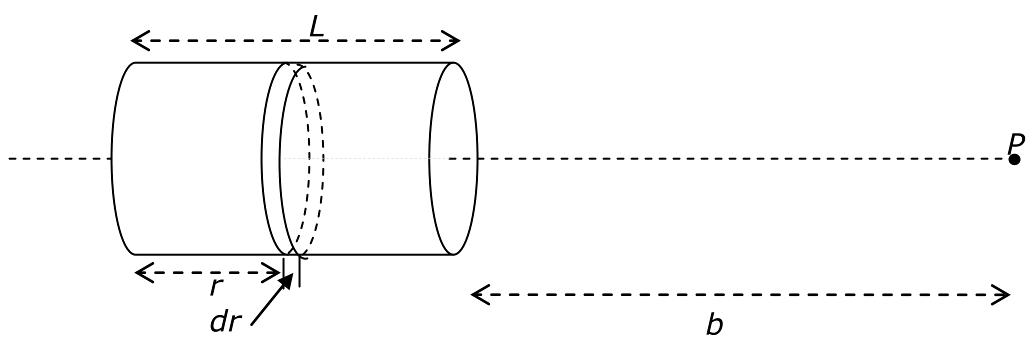
Consider an infinitely thin ring of width dr at a distance r from the left edge of the cylinder. The surface area of this ring element will be $2\pi a*dr$

As the charge per unit area on the cylinder is σ, the charge on the ring is given by

$dQ = 2\pi a*dr*\sigma$ ------------------------- (1)

The distance of point P on the axis of this ring element from the center of the ring will be

$x = L + b - r$ --------------------------- (2)



The field due to a uniformly charged ring of radius a at a distance x from its center on its axis is given by

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q\,x}{(x^2+a^2)^{3/2}}$$

Substituting the values for the ring element, the field due to this element dE at point P is given by

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ\,x}{(x^2+a^2)^{3/2}}$$

Or $$dE = \frac{1}{4\pi\epsilon_0}\frac{2\pi a*dr*\sigma*(L+b-r)}{[(L+b-r)^2+a^2]^{3/2}}$$

Or $$dE = \frac{2\pi a\sigma}{4\pi\epsilon_0}\frac{(L+b-r)\,dr}{[(L+b-r)^2+a^2]^{3/2}}$$

Thus the field at P due to the whole cylinder is given by adding the fields due to all such ring elements i.e. by integrating the field due to this ring element as

$$E = \int dE = \frac{2\pi a\sigma}{4\pi\epsilon_0} \int_0^L \frac{(L+b-r)\, dr}{[(L+b-r)^2+a^2]^{3/2}}$$

Now put $(L + b - r)^2 + a^2 = u^2$

Differentiating we get $2(L + b - r)(-dr) = 2u * du$

When r = 0, u = u_1 = $\sqrt{(L + b)^2 + a^2}$

And when r = L, u = u_2 = $\sqrt{b^2 + a^2}$

Su*bstituting these, the integral reduces to*

$$E = \frac{2\pi a\sigma}{4\pi\epsilon_0} \int_{u_1}^{u_2} \frac{-u*du}{(u^2)^{3/2}}$$

Or $$E = -\frac{2\pi a\sigma}{4\pi\epsilon_0} \int_{u_1}^{u_2} \frac{du}{u^2}$$

Or $$E = -\frac{2\pi a\sigma}{4\pi\epsilon_0} \int_{u_1}^{u_2} u^{-2}\, du$$

Or $$E = -\frac{2\pi a\sigma}{4\pi\epsilon_0} \left(-\frac{1}{u}\right)_{u_1}^{u_2}$$

Or $$E = \frac{2\pi a\sigma}{4\pi\epsilon_0} \left(\frac{1}{u_2} - \frac{1}{u_1}\right)$$

Substituting values of u_1 and u_2 from above we get the expression for the field at P due to the cylinder as

$$E = \frac{2\pi a\sigma}{4\pi\epsilon_0} \left(\frac{1}{\sqrt{b^2+a^2}} - \frac{1}{\sqrt{(l+b)^2+a^2}}\right)$$

Or $$E = K * 2\pi a\sigma \left(\frac{1}{\sqrt{b^2+a^2}} - \frac{1}{\sqrt{(l+b)^2+a^2}}\right)$$

This is the required expression.