Q- A boat is trying to get from point A to point B, which is exactly to the east of A. The ocean current is due north at 4.8 km/h. The boat speed relative to the water is 18 km/h.

Reading:

The most common confusion in this type of questions is due to the word relative velocity.

As we know that if a body A, moving with velocity $\vec{v}_A$ and another body B moving with velocity $\vec{v}_B$, then the velocity of body A relative to the velocity of body B is given by

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

But when the word velocity of the boat relative to water is given, it means the velocity of the boat in still water. As water is also moving, the current is also dragging the boat with its velocity and hence the actual velocity of the boat (relative to ground) is the resultant velocity of the boat due to the rowing and driven by the current both.

If the velocity of the boat in still water (only due to rowing) is $\vec{v}_B$ and the velocity of the current is $\vec{v}_w$ then actual velocity of the boat in the current relative to ground will be given by

$$\vec{v} = \vec{v}_B + \vec{v}_w$$

This may also be understood as

Velocity of boat relative to water = velocity of boat - velocity of water

Or $\vec{v}_B = \vec{v} - \vec{v}_w$

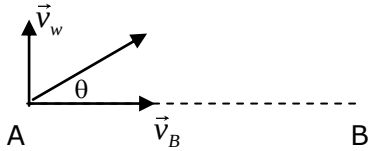
Gives $\vec{v} = \vec{v}_B + \vec{v}_w$

Solutions:

A) if the captain heads his boat due east, what will be his actual course relative to the shore?

Velocity of the boat in still water

$$\vec{v}_B = 18.0 * \hat{i}$$



Velocity of current

$$\vec{v}_w = 4.8 * \hat{j}$$

Where $\hat{i}$ and $\hat{j}$ are unit vectors in x (east) and y (north) directions respectively

Hence the resultant velocity of the boat is given by the resultant of the two velocities as

$$\vec{v} = \vec{v}_B + \vec{v}_w$$

Or $\vec{v} = 18.0 * \hat{i} + 4.8 * \hat{j}$

The magnitude of this velocity will be given by

$$|\vec{v}| = \sqrt{18.0^2 + 4.8^2} = \sqrt{324 + 23.04} = 18.63 \text{ km/h}$$

And the direction is given by

$$\tan\theta = \frac{4.8}{18} = 0.2667$$

Gives $\theta = 14.93^0$

Hence the boat is heading **14.93^0 north of east** with speed of **18.63 km/h**.

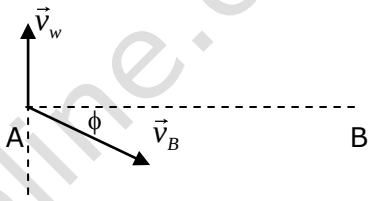
b) To attain a straight course, due east, what should be the heading of the boat?

To reach to point the boat should rowed in such a direction that the resultant velocity of the boat will be towards east.

For this the velocity of the current is to be nullify, hence the component of the velocity of the boat must be equal and opposite to the velocity of water and thus the boat must be heading to south east, say at an angle ϕ south of the east then

$$|\vec{v}_B|\sin\phi = |\vec{v}_w|$$

Or $$\sin\phi = \frac{|\vec{v}_w|}{|\vec{v}_B|} = \frac{4.8}{18.0} = 0.2667$$



Gives ϕ = **15.47^0**

The velocity with which it moves to the ease will be $|\vec{v}_B|\cos\phi = 17.35$ km/h.

To go straight from B to A (reversed) the direction will be $\phi = 15.47^0$ south of west.