Q- A billiard ball of mass m rolls without slipping across a table at speed v_0. It collides with the lower end of a rod of mass M = 2m and length d, pivoted at its center and initially at rest in vertical position. After collision the ball rolls straight with half of its initial speed. What is the rod's angular velocity after the collision?

When a ball rolls without slipping with velocity v_0, it has angular velocity as well, related to the translational velocity as

$$\omega_0 = \frac{v_0}{R}$$

Here R is the radius of the ball.

The ball will collide normally to the rod. Let the force on the rod due to the ball is F acting for a time dt, and thus applying impulse F dt on the rod.

The impulsive torque on the rod about its center will be F dt (d/2) and as the impulsive torque is equal to the change in angular momentum, the angular velocity ω of the rod after collision is given by the relation

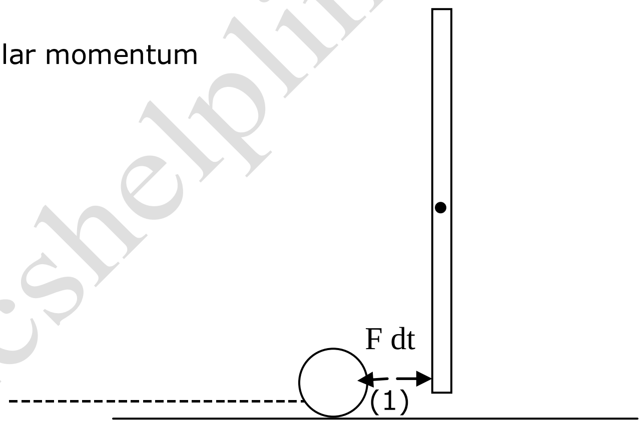
Impulsive torque = change in angular momentum

Or $$F\ dt * \frac{d}{2} = I_R \omega$$

Or $$\omega = \frac{F\ dt}{I_R} * \frac{d}{2}$$

Or $$\omega = \frac{F\ dt}{(Md^2/12)} * \frac{d}{2}$$

Or $$\omega = \frac{6\ F\ dt}{Md}$$



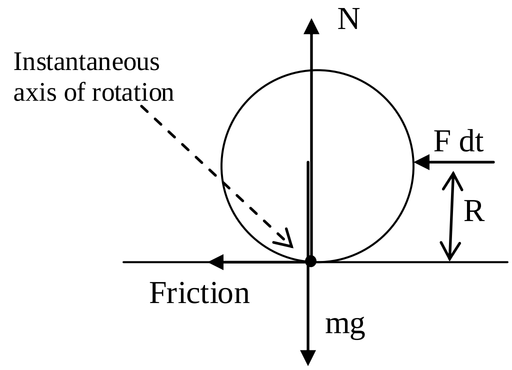
As during impact, ball's velocity decreases to half its original velocity, the same will be with its angular velocity. This will be done by the reactionary impulsive torque of the rod and friction between the ground and the ball. The friction acting on the ball during time of impact is unknown we cannot find the torque due to it about the center of the ball.

In pure rolling, the ball is translating and rotating about its center. The motion is such that at any instant, it may be considered as having pure rotation about the axis passing through the point of contact of the ball with the ground (called instantaneous axis of rotation). If we consider the motion as rotation about instantaneous axis of rotation then the torque due to friction becomes zero and the impulsive torque on the ball will be – F dt R

Hence during collision considering instantaneous axis of rotation we get

Impulsive torque = change in angular momentum

$$-\,F\ dt\ R = I_B\left(\frac{\omega_0}{2}\right) - I_B \omega_0$$



Or $\quad - F\ dt\ R = -\ I_B \left(\frac{\omega_0}{2}\right)$

Or $\quad - F\ dt\ R = -\ \left(\frac{2}{5}\ mR^2 +\ mR^2\right)\left(\frac{\omega_0}{2}\right)$

(Moment of inertia of the ball about instantaneous axis of rotation is calculated using parallel axis theorem.)

Or $\quad - F\ dt\ R = -\ \frac{7}{10}\ mR^2\omega_0$

Or $\quad F\ dt = \frac{7}{10}\ mR\ \omega_0$

Substituting this value of F dt in equation (1) we get

$$\omega = \frac{6}{Md}\left(\frac{7}{10}\ mR\ \omega_0\right)$$

Or $\quad \omega = \frac{6}{2m\ d}\left(\frac{7}{10}\ mR\ \omega_0\right)$

Or $\quad \omega = \frac{21\ R}{10\ \ d}\ \omega_0$

(As there may be some sliding at the time of collision and the collision is not told to be elastic, the total mechanical energy will not be conserved.)