Q- A small block of mass 80 g is oscillating on a spring. The equation of motion for its position x as a function of time is given by

x(t) = (5.0 cm) cos (10t + π/4)

(a) what is the amplitude of the oscillation?
(b) What is the period of oscillation?
(c) What is the position of the block at t = 5 sec?
(d) What is the velocity of the block at t = 5 sec?
(e) What is the maximum velocity of the block?
(f) What is the maximum potential energy of the system?
(g) What is the acceleration of the block at t = 5 sec?

The equation of motion is given by

$$x(t) = (5.0\ cm)\cos(10t + \pi/4)$$

Comparing it with standard equation of simple harmonic motion

$$x(t) = A\cos(\omega t + \emptyset_0)$$

(a) The amplitude A = 5.0 cm = 0.050 m
(b) The angular frequency ω = 10 rad/s
And the initial phase $\emptyset_0 = \pi/4$ rad

Hence the period of oscillation is given by

$$T = \frac{2\pi}{\omega} = \frac{6.28}{10} = 0.628\ s$$

c) The position of the mass at t = 5 s is given by

$$x(5) = (5.0\ cm)\cos(10 * 5 + \pi/4)$$

Or $$x(5) = (5.0\ cm)\ cos\left(50 * \frac{\pi}{3.14} + \frac{\pi}{4}\right)$$

Or $$x(5) = (5.0\ cm)\ cos(15.9\ \pi + 0.25\ \pi)$$

Or $$x(5) = (5.0\ cm)\ cos(16.17\ \pi)$$

Or $$x(5) = (5.0\ cm)\ cos(16\ \pi + 0.17\ \pi)$$

Or $$x(5) = (5.0\ cm)\ cos(0.17\ \pi)$$

Or $$x(5) = (5.0\ cm)\ cos(30.6)^0$$

Or $$x(5) = (5.0\ cm)0.861 = 4.30\ cm$$

d) The velocity as a function of time is given by

$$v(t) = \frac{d\ x(t)}{dt} = (5.0\ cm)\frac{\mathrm{d}}{\mathrm{dt}}\cos(10t + \pi/4)$$

Or $$v(t) = -(50.0\ cm/s)\sin(10t + \pi/4)$$

Hence the velocity of the particle at t = 5 s will be

$$v(5) = -(50.0\ cm/s)\sin(30.6^0)$$

Or $v(5) = -50 * 0.509 = -25.4\ cm/s = -0.254\ m/s$

e) The maximum velocity of the particle is given by

$$v_{max} = A\omega = 5.0 * 10 = 50\ cm/s = 0.5\ m/s$$

f) The maximum kinetic energy of the system will be

$$\frac{1}{2}mv^2{}_{max} = 0.5 * 0.08 * (0.5\ m)^2 = 0.01\ J$$

Maximum potential energy of the system with respect to the equilibrium position will be same as that of the kinetic energy and hence equal to 0.01 J.

g) The acceleration is the rate of change of velocity and thus given by

$$a = \frac{dv(t)}{dt} = -(50.0\ cm/s)\ \frac{d}{dt}\sin(10t + \pi/4)$$

Or $a = \frac{dv(t)}{dt} = -(500.0\ cm/s^2)\ \cos(10t + \pi/4)$

Or $a(t) = -(5.0\ m/s^2)\ \cos(10t + \pi/4)$

Thus $a(5) = -(5.0\ m/s^2)\ \cos(10 * 5 + \pi/4)$

Or $a(5) = -(5.0\ m/s^2)\ \cos(30.6^0) = 4.3\ m/s^2$