Q- A 65.0kg boy and his 40.0 kg sister, both wearing roller blades, face each other at rest. The girl pushes the boy hard, sending him backward with a velocity 2.90 m/s toward the west. Ignore friction. (a) Describe the subsequent motion of the girl. (b) How much chemical energy is converted into mechanical energy in the girl's muscles?

The problem is based on the law of conservation of momentum. If there is no any external force acting on a system, its linear momentum is conserved.

(a)
If we consider the boy and the girl as a system then there is no any external force in horizontal direction. The force with which the girl pushes the boy and the reaction force on the girl by the boy (Newton's third law) are internal forces; hence the momentum of the system remains conserved. As the reaction force on the girl is in opposite direction the girl will be pushed backward and hence will move in backward direction.

Let the mass of the boy is m_1 = 65.0 kg and that of the girl is m_2 = 40.0 kg. the velocity with which the boy is pushed is v_1 = 2.90 m/s and the velocity of the girl after the push is v_2.

As both are initially at rest, initial momentum of the system will be zero.

Momentum of the system after push will be
Momentum of boy + momentum of girl = $m_1v_1 + m_2v_2$
According to law of conservation of momentum in horizontal direction

Final momentum = initial momentum

Or $m_1v_1 + m_2v_2 = 0$

Gives $v_2 = -\frac{m_1 v_1}{m_2} = -\frac{65.0 * 2.90}{40} = -4.71 \text{ m/s}$

Negative sign means that the velocity of the girl is in negative direction. Hence after push the girl will move towards east with a velocity of 4.71 m/s.

(b)

As initially both were at rest (zero kinetic energy) hence the kinetic energy of the system after push is acquired from the muscles energy of the girl.

Kinetic energy of the boy after push

$KE_1 = ½*m_1v_1^2 = 0.5*65.0*2.90^2 = 273.3$ J

Kinetic energy of the girl after push

$KE_2 = ½*m_2v_2^2 = 0.5*40.0*4.71^2 = 443.7$ J

Hence total energy of the system after push will be

$E = E_1 + E_2 = 273.3 + 443.7 = 717$ J

Hence the chemical energy converted to mechanical energy is 717 J.