

Q- A scuba diver releases a 2.50-cm-diameter (spherical) bubble of air from a depth of 15.0 m in a lake. Assume the temperature is constant at 16.0°C, and the air behaves as a perfect gas. What is the diameter of the bubble when it reaches the surface?

The volume of air in bubble at depth h will be

$$V_1 = \frac{4}{3} \pi \left(\frac{D_1}{2}\right)^3$$

The pressure at depth h will be

$$P_1 = P_{atm} + h\rho_w g$$

Pressure at the surface

$$P_2 = P_{atm}$$

If the diameter at the surface is D_2 then

$$V_2 = \frac{4}{3} \pi \left(\frac{D_2}{2}\right)^3$$

At constant temperature according to Boyle's law

$$P_2 V_2 = P_1 V_1$$

$$\text{Or } P_{atm} * \frac{4}{3} \pi \left(\frac{D_2}{2}\right)^3 = (P_{atm} + h\rho_w g) * \frac{4}{3} \pi \left(\frac{D_1}{2}\right)^3$$

$$\text{Or } P_{atm} * (D_2)^3 = (P_{atm} + h\rho_w g) * (D_1)^3$$

$$\text{Or } (D_2)^3 = \left(1 + \frac{h\rho_w g}{P_{atm}}\right) * (D_1)^3$$

$$\text{Or } D_2 = \left(1 + \frac{h\rho_w g}{P_{atm}}\right)^{1/3} * D_1$$

$$\text{Or } D_2 = \left(1 + \frac{15 * 1000 * 9.8}{1.03 * 10^5}\right)^{1/3} * 2.50 \text{ cm}$$

$$\text{Or } D_2 = \left(1 + \frac{1.74 * 10^5}{1.03 * 10^5}\right)^{1/3} * 2.50 \text{ cm}$$

$$\text{Or } D_2 = (2.4)^{1/3} * 2.50 \text{ cm} = 1.34 * 2.50 = 3.35 \text{ cm}.$$