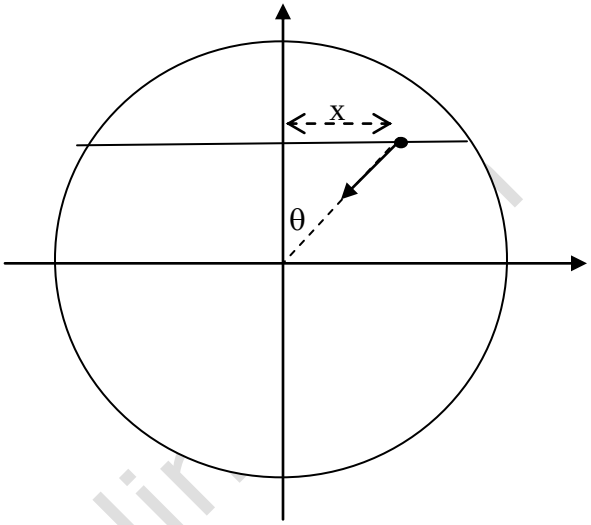
Q- A straight tunnel is dug through the earth (not passing through the center). Show that neglecting any resistive force the motion of an object inside the tunnel is simple harmonic motion and find the expression for time period of the motion.

As we know that the gravitational force on an object of mass m inside the earth at a distance r from the center of earth is given by

$$\vec{F}_r = -\left(\frac{GmM_E}{R_E^3}\right)\vec{r}$$

(Here M_E and R_E are mass and radius of earth respectively)



Or $F_r = -k\,r$ (say)

Component of this force along the tunnel will be

$$F_x = -k\,r\,sin\theta$$

The other component of the force is perpendicular to the tunnel and will be balance by the normal reaction of the tunnel as it can move only along the tunnel.

If the distance of the object from the midpoint of the tunnel be x (= r sin θ), then the component of the force along the tunnel is given by

$$F_x = -k\,x$$

As the force is restoring and as it is proportional to the distance from the midpoint of the tunnel which is the equilibrium position, the motion of the object is the simple harmonic motion.

The time period of the simple harmonic motion is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Substituting the value of the force constant k in this situation we get

$$T = 2\pi\sqrt{\frac{mR_E^3}{GmM_E}}$$

Or $$T = 2\pi\sqrt{\frac{R_E^3}{GM_E}}$$

This is the required expression for the time period of the object.