Q- Consider an object on which the net force is a resistive force proportional to the square of its speed as $F = -kmv^2$, where k is a constant and m is mass of the object. At t = 0 the speed of the object is Vo. Find the speed of the object at subsequent time t.

Let at time t velocity of the object is v.

According to Newton's law of motion we have

$$F = ma$$

As the force is variable, the acceleration can be given in differential form as (a = dv/dt) hence the above equation can be written as

$$\frac{dv}{dt} = \frac{F}{m}$$

As the net force on the skater is given by - kmv^2 we have

$$\frac{dv}{dt} = \frac{-kmv^2}{m}$$

Or $$\frac{dv}{dt} = -k * v^2$$

This equation is a variable separable type of differential equation and hence we can write it as

$$\frac{dv}{v^2} = -k * dt \qquad \text{---------------} \ (1)$$

Integrating above equation with proper limits as time t is changing from 0 to t and the velocity is changing from v_0 to v. hence we get

$$\int_{V_0}^{v} \frac{dv}{v^2} = -k * \int_0^t dt$$

Or $$\left[\frac{v^{-2+1}}{-2+1}\right]_{V_0}^{v} = -k * [t]_0^t$$

Or $$\left[\frac{-1}{v}\right]_{V_0}^{v} = -k * [t]_0^t$$

Or $$\left[\frac{-1}{v} - \frac{-1}{V_0}\right] = -k * [t - 0]$$

Or $\frac{1}{v} = \frac{1}{V_0} + kt$

Or $\frac{1}{v} = \frac{1+V_0kt}{V_0}$

Gives $v = \frac{V_0}{1+V_0kt}$

Hence velocity of the skater as a function of time is given by

$v(t) = \frac{V_0}{1+V_0kt}$ Hence shown