Q- Two particles executes SHM of same amplitude of 20 cm with same period along the same line and about same equilibrium position. If the maximum distance between them is 20 cm, find their phase difference.

Let at time t their displacements from the mean position are given by

$x_1 = A\sin\omega t$

And $\quad x_2 = A\sin(\omega t + \varphi)$

Here ϕ is the phase difference

Thus $\quad \Delta x = x_2 - x_1 = A\sin(\omega t + \varphi) - A\sin\omega t$

Or $\quad \Delta x = A[\sin(\omega t)\cos\varphi + \cos\omega t\sin\varphi - \sin\omega t] \qquad (1)$

For Δx to be maximum or minimum $\Delta x/dt$ should be zero thus differentiating we get

$$\left(\frac{d}{dt}\right)\Delta x = A\omega[\cos\omega t)\cos\varphi - \sin\omega t\sin\varphi - \cos\omega t] = 0$$

Or $\quad \cos\omega t\cos\varphi - \sin\omega t\sin\varphi - \cos\omega t = 0$

Or $\quad \cos\omega t\,(\cos\varphi - 1) - \sin\omega t\sin\varphi = 0$

Gives $\tan\omega t = \frac{\cos\varphi - 1}{\sin\varphi}$

Thus $\quad \sin\omega t = \frac{\tan\omega t}{\sqrt{1+tan^2\,\omega t}} = \frac{\cos\varphi - 1}{\sqrt{sin^2\varphi + cos^2\varphi - 2\cos\varphi + 1}}$

Or $\quad \sin\omega t = \frac{\cos\varphi - 1}{\sqrt{2(1-\cos\varphi)}} = -\frac{1}{\sqrt{2}}\sqrt{1-\cos\varphi} = -\frac{1}{\sqrt{2}}\sqrt{2\sin^2\left(\frac{\varphi}{2}\right)}$

Or $\quad \sin\omega t = -\sin\left(\frac{\varphi}{2}\right)$

And $\quad \cos\omega t = \cos\left(\frac{\varphi}{2}\right)$

Substituting in equation (1) the maximum distance between the two particles is given by

$$\Delta x_{max} = A\left[-\sin(\frac{\varphi}{2})\cos\varphi + \cos\left(\frac{\varphi}{2}\right)\sin\varphi + \sin\left(\frac{\varphi}{2}\right)\right]$$

Or $\quad \Delta x_{max} = 2A\sin\left(\frac{\varphi}{2}\right)$

Substituting the numerical values, we get

$$0.20 = 2 * 0.20\ \sin\left(\frac{\varphi}{2}\right)$$

Or $\quad \sin\left(\frac{\varphi}{2}\right) = 1/2$

Or $\quad \phi = 2\pi/3$