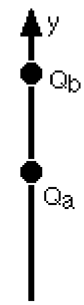
Q- A rigid, insulating fiber if fixed vertically. A charge Q_a = -7 μC is fixed to the fiber at some point. A bead with a hole drilled through its center is slipped over the fiber above Q_a and is free to move along the fiber without friction. The mass of the bead is *m* = 160 g and its charge is Q_b. At equilibrium, the bead floats a distance y_b = 16 cm above the Q_a.

(a) Calculate the charge on the bead.



The force between two point charges Q_1 and Q_2 at a distance r is given by Coulomb's law and the magnitude of which is given as

$$F = \frac{Q_1 Q_2}{4\pi \in_0 r^2}$$

Here $\in_0$ is the permittivity of free space

The bead will remain in equilibrium means that the electrostatic force between the two charges balances the weight of the bead and for that the two forces are equal in magnitude and opposite in the direction. Hence we have

$$\frac{Q_a Q_b}{4\pi \in_0 {y_b}^2} = mg$$

Here m is the mass of the bead and g is acceleration due to gravity.

Now as $\frac{1}{4\pi \in_0} = 9*10^9$ Nm^2/C^2, substituting the values in above equation we get

$$\frac{9*10^9 *(-7*10^{-6} C)Q_b}{(0.16m)^2} = (0.160kg)*9.81$$

Gives $Q_b = \frac{0.160*9.81*(0.16)^2}{9*10^9 *(-7*10^{-6})} = -6.378*10^{-7} C$

Hence the charge on the bead must be

Q_b = - 6.378*10^{-7} C

(b) If the mass of the bead were tripled (*m*' = 3*m*), how far above the origin would it now float at equilibrium?

If the mass of the bead is tripled to balance the new weight the distance between the charges must be reduced and hence considering the new equilibrium we have

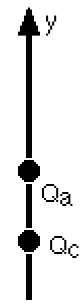
$$\frac{Q_a Q_b}{4\pi \in_0 (y_b')^2} = m' g$$

Or $$\frac{9*10^9 *\left(-7*10^{-6}\right)\left(-6.378*10^{-7}\right)*}{(y_b')^2} = \left(3*0.160\right)*9.81$$

Gives $$(y_b')^2 = \frac{9*10^9 *\left(-7*10^{-6}\right)\left(-6.378*10^{-7}\right)*}{\left(3*0.160\right)*9.81} = 8.533*10^{-3}$$

Or yb' = 0.0924 m

y_b' = 9.24 cm



(c) The bead (with original mass *m*) is removed and its charge altered to *Qc*. When placed on the fiber from below Qc, it floats at equilibrium point *yc* = -11 cm below the origin. What is this new charge?

Now as the bead is below the origin the force between the two charges must be attractive so that the weight of the bead will be balanced and the new equilibrium is reached. Hence we have

$$-\frac{Q_a Q_c}{4\pi \in_0 (y_c)^2} = mg$$

Or $$Q_c = -\frac{mg * 4\pi \in_0 (y_c)^2}{Q_a} = -\frac{0.160 * 9.81 * (0.11)^2}{9*10^9 * (-7*10^{-6})} = 3.015*10^{-7} C$$

$\boldsymbol{Q_c = 3.015*10^{-7}}$ **C**