Q- (a) When the displacement of a mass on a spring is one-sixteenth of *A*, what fraction of the energy is kinetic and what fraction is potential?

The total energy of oscillating system is given by

$U_T = (1/2)\ KA^2$

When the displacement of the mass from equilibrium position is x its potential energy Which is stored in the spring as elastic potential energy is given by

$U_P = (1/2)\ Kx^2$

And hence the kinetic energy at that position is given by

$U_K = U_T - U_P = (1/2)\ KA^2 - (1/2)\ Kx^2 = (1/2)\ K(A^2 - x^2)$

Thus the fraction of the kinetic energy will be

$$f_K = \frac{U_K}{U_T} = \frac{(1/2)K\left(A^2 - x^2\right)}{(1/2)KA^2} = \frac{\left(A^2 - x^2\right)}{A^2} = 1 - \left(\frac{x}{A}\right)^2$$

But from the question x = A/16 hence or x/A = 1/16 hence

$$f_K = 1 - \left(\frac{x}{A}\right)^2 = 1 - \frac{1}{256} = \frac{255}{256} = 0.996$$

And in percentage will be 0.996*100 = **99.6 %**

The potential energy fraction will be

$$f_P = \frac{U_P}{U_T} = \frac{(1/2)Kx^2}{(1/2)KA^2} = \frac{x^2}{A^2} = \left(\frac{x}{A}\right)^2$$

Or $f_P = 1/256$

And in percentage (1/256)*100 = **0.39 %**

(b) At what displacement, as a fraction of *A*, is the energy one-eighth kinetic and seven-eighths potential?

As above the fraction of kinetic energy

$$f_K = 1 - \left(\frac{x}{A}\right)^2 = \frac{1}{8}$$

Or $$\left(\frac{x}{A}\right)^2 = 1 - \frac{1}{8} = \frac{7}{8}$$

Or $$\frac{x}{A} = \sqrt{\frac{7}{8}} = 0.935$$

Or x = **0.935 A**