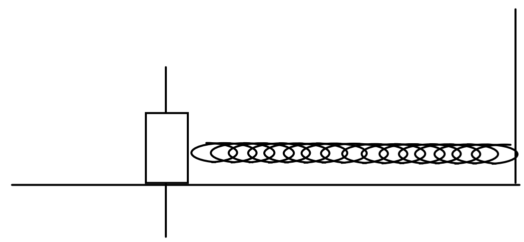
Q- An object of mass m is traveling on a horizontal surface. There is a coefficient of kinetic friction μ between the object and the surface. The object has speed v when it reaches x = 0 and encounters a spring. The object compresses the spring, stops, and then recoils and travels in the opposite direction. When the object reaches x = 0 on its return trip, it stops. Find k, the spring constant in terms of other given quantities.

The problem is based on the work energy rule. According to work energy rule the work done on the system is equal to increase in its energy.



There are two force acts on the object

1. The friction and
2. The force of the spring.

The friction and spring force both initially opposite the direction of motion and hence work done by both is negative.

As the friction is a non-conservative force the work done against it converts in heat energy. The spring force is the conservative one and hence work done against it is remains stored in it in form of elastic potential energy.

The magnitude of friction force is given by the product of the normal force of the surface and the coefficient of sliding friction. The normal force is balancing the weight mg of the object hence

$N = mg$

And $F = \mu * mg$

If the displacement against the friction is x, then the work done by the friction is given by

$W_f = - \mu * mg * x$

The work done against the spring remains stored in it in form of its elastic potential energy and is given by

$Ws = ½ Kx^2$

Where K is the spring constant and x is the compression or elongation in the spring.

Now let the object first comes to rest after moving x, means the spring is compressed by x.

Hence in this motion

Work done by the friction $W_f = - \mu * mg * x$

The work done by spring $Ws = - ½ Kx^2$

According to the work energy rule

Work done = increase in kinetic energy

Or Wf + Ws = = final KE - initial KE

Or $- \mu * mg * x - ½ Kx^2 = 0 - ½ mv^2$

Or $\mu * mg * x + ½ Kx^2 = ½ mv^2$ --------------------------- (1)

When the object recoils and comes back to the original point x = 0, [compression in spring = 0, the stored energy utilized to do work]

Work done by friction $W_f' = - \mu * mg * x$

Work done by the spring $Ws' = ½ Kx^2$

[As the spring force is in the direction of motion]
Hence applying work energy rule for the recoil journey

Or Wf' + Ws' = final KE - initial KE

Or $- \mu * mg * x + ½ Kx^2 = 0 - 0 = 0$ ------------------------------------ (2)

Subtracting equation (2) from (1) we get

$$2 \mu * mg * x = ½ mv^2$$

Gives $x = \dfrac{v^2}{4\mu g}$ ------------------------------------ (3)

From equation (2) we have

$$-\mu mgx + \frac{1}{2}kx^2 = 0$$

Or $K = \dfrac{2\mu mg}{x}$ [x is not equal to zero]

Substituting the value of x from equation (3) we get

$$K = \frac{8\mu^2 mg^2}{v^2}$$