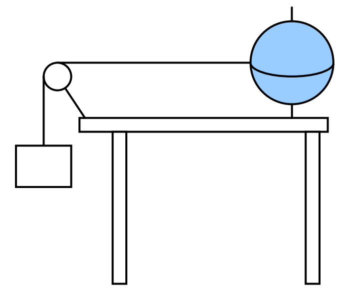
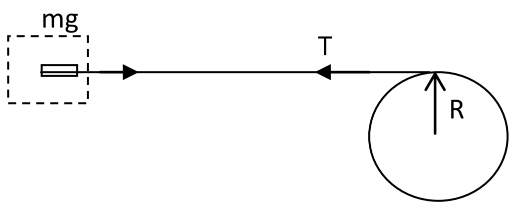
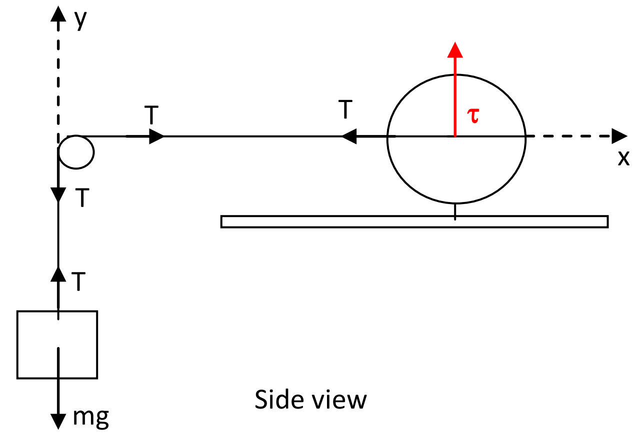
Q- A globe of mass M and radius R can rotate about its fixed vertical axis. A block of mass m is attached by a massless string/pulley system as shown. As the block is released, it causes the globe to spin. If the block starts at rest, at what speed does the block hit the ground after it falls a distance h?



Method 1: Using force and torque

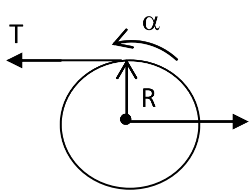


Top view

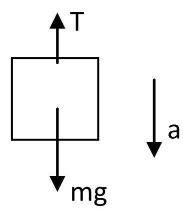


Side view

FBD for the globe



FBD for the block



There will be torque acting on the globe due to the tension in the string. As due to this torque the globe will rotate in CCW direction seen from the top, according to right hand thumb rule, the direction of this torque vector will out of the page i.e. vertically upward.

As the block accelerates downwards under the tension and the weight, the equation of motion can be written as

$$T - mg = -ma \qquad \text{------------------------------------- (1)}$$

As the tension in the string acts on the globe tangentially and the radius of the globe is R the magnitude of the torque acting on it will be T*R and if α is the angular acceleration of the globe, the equation of rotational motion can be written as

$$\tau = I * \alpha$$

Or $$T * R = \frac{2}{5}MR^2 * \alpha$$

Or $\quad T = \frac{2}{5} M\ R * \alpha$ -- (2)

As the length of the string remains unchanged, the acceleration of the block and the string will be the same as the acceleration of the point on the equator of the globe and thus we have

$$a = \alpha * R$$ -- (3)

Substituting values of T from equation (1) and α from equation (3) in equation (2) we get

$$mg - ma = \frac{2}{5} M\ R * \frac{a}{R}$$

Or $\quad mg\ = \frac{2}{5} M\ a + ma$

Or $\quad a = \frac{5mg}{2M+5m}$

This is the acceleration (constant) of the block and thus the velocity v as it falls distance h is given by second equation of motion as

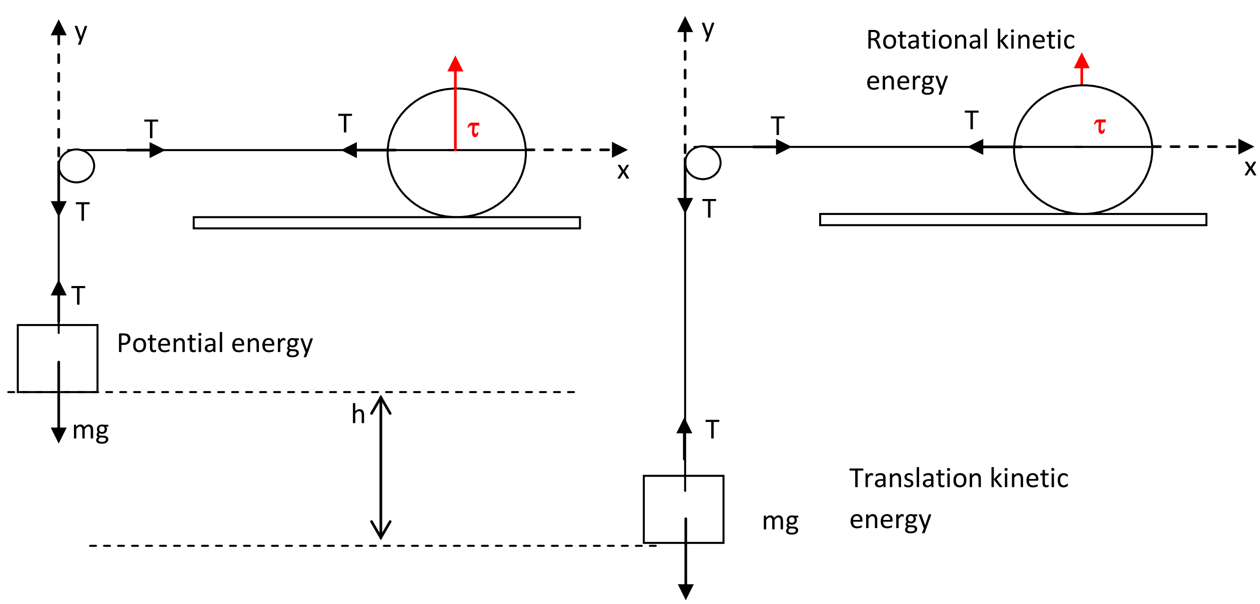
$$[v^2 = u^2 + \ 2as]$$

$$v^2 = 0 + 2 * a * h$$

Or $\quad v = \sqrt{2\left(\frac{5mg}{2M+5m}\right)h}$

Or $\quad v = \sqrt{\frac{10mgh}{2M+5m}}$

Method 2: Using energy conservation law.



Here we can use the conservation of energy as there is no any non-conservative force acting on the system.

When the system is released the block and the globe are at rest so there is no kinetic energy in the system. The block is at a height h from the ground level thus with respect of the ground the block will have a gravitational potential energy of mgh.

When the system is released the block will accelerates downwards and the tension in the string rotates the globe. The block loses its potential energy, gets the translational kinetic energy and the globe gets rotational kinetic energy.

Thus according to law of conservation of energy

Initial energy = final energy

Or potential energy of the block = translational KE of the block + rotational KE of the globe

Or $mgh = \frac{1}{2}\, mv^2 + \frac{1}{2}\, I\omega^2$

Here v is the velocity of the block and w is the angular velocity of the globe. Related as $v = \omega R$ and I is the moment of inertia of the globe given by $I = \frac{2}{5}\, MR^2$

Thus substituting the values in equation above we get

$$mgh = \frac{1}{2}\, mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2$$

Or $\mathbf{2}mgh = mv^2 + \left(\frac{2}{5}M\right)v^2$

Or $\mathbf{10}mgh = (5\,m + 2M)\,v^2$

Gives $v = \sqrt{\frac{10mgh}{2M+5m}}$