

Q- A 240 g mass suspended by a 0.8 m long string is pulled 8.4° to one side and released. How long does it take for the pendulum to reach 6.0° on the opposite side?

For small angles the motion of a pendulum bob is simple harmonic with time period T given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Where L is the length of the pendulum

The angular frequency for this motion is given by

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{g}{L}} = 3.5 \text{ rad/sec.} = 200.6^\circ/\text{s}$$

The displacement from equilibrium position as a function of time is given by

$$X = \theta L = A \sin(\omega t + \phi)$$

Initially at $t = 0$ the bob is released from extreme position $\theta = \theta_{\max}$, the initial phase ϕ is 90° and hence the equation can be written as

$$\theta L = \theta_{\max} L \sin(\omega t + 90^\circ) \quad (\text{For small angles } \sin \theta = \theta \text{ in radians})$$

$$\text{Or } \theta = \theta_{\max} \cos(\omega t) \quad \text{----- (1)}$$

$$\text{Now here } \theta_{\max} = 8.4^\circ$$

$$\text{And } \theta = -6.0^\circ$$

Substituting in equation 1 we get

$$-6.0^\circ = 8.4^\circ \cos[(200.6) \cdot t]$$

$$\text{Or } \cos[(200.6) \cdot t] = -6.0^\circ/8.4^\circ = -0.714$$

$$\text{Gives } (200.6) \cdot t = \cos^{-1}(-0.714) = [135.58^\circ]$$

$$\text{Gives } t = 135.58/200.6 = 0.676 \text{ s}$$

