Q- A 150 m long rope of mass 250 kg is wrapped on a spool and its 50.0 m is hanging vertically. Now the spool is rotated to pull the rope up slowly. How much work is to be done to pull further 20.0 m of the rope? Neglect any friction force.

Let the mass of the rope is M and its length is L.

The mass of the rope per unit length is M/L

Let the length of the hanging part be *l* initially and it is pulled further by x.

The mass of the hanging part of the rope initially will be given by

$$m_1 = \frac{M}{L} * l$$

The depth of the center of mass of this hanging part will be *l*/2 and thus its potential energy relative to the spool position is given by

$$U_1 = m_1 g\left(-\frac{l}{2}\right) = -\frac{M}{L} * l\,\mathrm{g}\left(\frac{l}{2}\right) = -\frac{Mgl^2}{2L}$$

And similarly the mass after the x more length is pulled up will be

$$m_2 = \frac{M}{L} * (l - x)$$

The depth of the center of mass of this part will be (*l*- x)/2 and thus its potential energy relative to the spool position is given by

$$U_2 = m_2 g\left(-\frac{l-x}{2}\right) = -\frac{M}{L} * (l - x)g\,\frac{(\mathrm{l}-\mathrm{x})}{2} = -\frac{Mg(l-x)^2}{2L}$$

Hence using work energy rule

Work done = increase in potential energy

Or $W = U_2 - U_1$

$$W = -\frac{Mg(l-x)^2}{2L} - \left(-\frac{Mgl^2}{2L}\right)_1$$

Or $W = \frac{Mg}{2L}[-(l-x)^2 + l^2]$

Or $W = \frac{Mg}{2L}[2lx - x^2]$

Or $W = \frac{Mgx}{2L}[2l - x]$

Substituting numerical values we get

$$W = \frac{250*9.8*20}{2*150}[2 * 50 - 20] = 1.31 * 10^4\,J$$