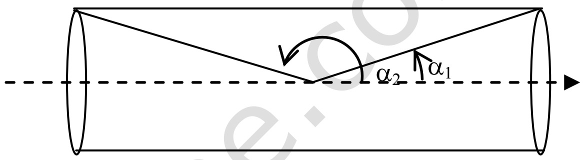
Q- A solenoid has length L = 30 cm, radius 6 cm, and N_1 = 5500 turns and its axis coincides with the z-axis. A small circular conducting loop containing N_2 = 15 turns of radius a = 3 cm is **centered** inside the solenoid; the plane of the loop makes a 30° angle with respect to the z-axis. The loop has a net resistance of R = 0.015 Ω. The current in the windings of the solenoid is varying with time according to the expression $I_1(t)$ = 0.35 A + 0.75 (A/s) t. Calculate the induced current in the loop at t = 4 s.

The number of turns in the solenoid N_1 = 5500
The length of the solenoid L = 30 cm = 0.3 m
Hence the number of turns per unit length will be

n = N_1/L = 55000/3

The magnetic field B in a solenoid carrying current I_1 at point on its axis, where the edges of the solenoid subtends angles α_1 and α_2 is given by

$$B = \frac{1}{2}\mu_0 n I_1\left(\cos\alpha_1 - \cos\alpha_2\right)$$



Or $$B = \frac{\mu_0 N_1 I_1}{2L}\left(\cos\alpha_1 - \cos\alpha_2\right)$$

As the loop is **centered** at the solenoid, the angle $\alpha_2 = 180^0 - \alpha_1$ and hence the field at the center of the solenoid is given by

$$B = \frac{\mu_0 N_1 I_1}{2L}\left(\cos\alpha_1 - \cos(180^0 - \alpha_1)\right) = \frac{\mu_0 N_1 I_1}{2L}\left(\cos\alpha_1 + \cos\alpha_1\right) = \frac{\mu_0 N_1 I_1 \cos\alpha_1}{L}$$

And the flux through the loop at the center is given by

$$\phi_B = N_2\left(\vec{B}\bullet\vec{A}\right) = N_2\left(\frac{\mu_0 N_1 I_1 \cos\alpha_1}{L}\right)A\cos\theta = \frac{\mu_0 N_1 N_2 A I_1 \cos\alpha_1 \cos\theta}{L}$$

Where θ is the angle between the magnetic field vector and the area vector A (I think indicated by Z' in the diagram.

As the area vector is normal to the plane of the loop the angle between the field B and the area A vectors will be $\theta = 90^0 + 30^0 = 120^0$

Hence the EMF induced in the loop is given by faraday's law as

$$\varepsilon = -\frac{d\phi_B}{dt} = -\frac{\mu_0 N_1 N_2 A\cos\alpha_1 \cos\theta}{L} * \frac{dI_1}{dt}$$

Now as $I_1(t)$ = 0.35 A + 0.75 (A/s) t

$$\frac{dI_1}{dt} = 0 + 0.75 = \frac{3}{4}\text{ A/s}$$

(Rate of change of current is independent of time)

Substituting in above equation we get

$$\varepsilon = -\frac{d\phi_B}{dt} = -\frac{3\mu_0 N_1 N_2 A\cos\alpha_1 \cos\theta}{4L}$$

And the magnitude of the current in the loop as a function of time will be given by

$$I_2 = \frac{\varepsilon}{R} = \frac{3\mu_0 N_1 N_2 A \cos\alpha_1 \cos\theta}{4LR}$$

Now here

$\mu_0 = 4\pi*10^{-7}$
$N_1 = 5500$
$N_2 = 15$
$A = \pi r^2 = 3.14*0.03^2 = 2.83*10^{-3}$ m^2
$\text{Cos } \alpha_1 = \frac{15}{\sqrt{15^2+6^2}} = 0.9285$
$\text{Cos } \theta = \cos 120^0 = -0.5$
t = 4 s
L = 0.3 m
And R = 0.015 Ω

Substituting all data we get the current at t = 4 s as

$$I_2 = \frac{3*4\pi*10^{-7}*5500*15*2.83*10^{-3}*0.9285*(-0.5)}{4*0.3*0.015}$$

Or $I_2 = -0.0227$ A

The negative sigh is according to the direction of the current (Lenz Law)

Hence current in the loop at any time is **I = - 0.0227 A**