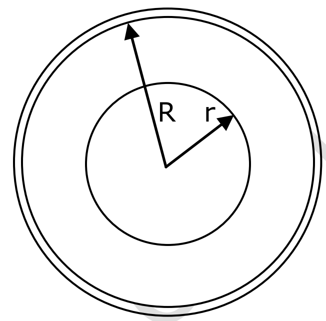
Q- A car tire of inner radius 16.5 cm and outer radius 30.5cm is modeled as having two sidewalls of uniform thickness 0.635cm and a tread wall of uniform thickness 2.50cm and width 20.0cm over it. Assume the rubber has uniform density equal to 1.10 x 10^3 kg/m^3, find its moment of inertia about an axis through its center.



The moment of inertia of the tire is the sum of the moment of inertia of the side walls and the thread wall.

The side walls can be considered as a uniform disc with a circular disc removed at the center.

Inner radius of the side wall r = 16.5 cm = 0.165 m

Outer radius of the side wall R = 30.5 cm = 0.305 m

Thickness of the side walls t = 0.635 cm = $6.35*10^{-3}$ m

Mass per unit area of the side wall will be given by the product of volume of unit area and density. The volume of unit area is numerically equal to the thickness and hence for the side walls the surface density is given by

$$\sigma = t * \rho = 6.35*10^{-3}*1.10*10^{3} = 6.985 \text{ kg/m}^2$$

The moment of inertia of a uniform disc of mass m and radius R is given by

$$I = \frac{1}{2}mR^2 = \frac{1}{2}(\sigma A)R^2 = \frac{1}{2}(\sigma\pi R^2)R^2 = \frac{1}{2}\sigma\pi R^4$$

Hence the moment of inertia of a side wall will be given by

MI of the whole disk of radius R - MI of the inner disc removed.

$$= \frac{1}{2}\sigma\pi R^4 - \frac{1}{2}\sigma\pi r^4 = \frac{1}{2}\sigma\pi\left(R^4 - r^4\right)$$

Hence the moment of inertia of two side walls will be given by

$$I_1 = 2*\frac{1}{2}\sigma\pi\left(R^4 - r^4\right) = \sigma\pi\left(R^4 - r^4\right) = 6.985*3.1416*\left(0.305^4 - 0.165^4\right)$$

Or I_1 = 0.1736 kgm^2

Now the moment of inertia of the tread wall is given by moment of inertia of the hollow cylinder of inner radius of 30.5 cm and outer radius of 33.0 cm.

Inner radius of the cylinder R = 30.5 cm = 0.305 m

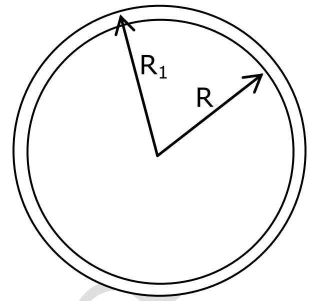
Outer radius of the cylinder R_1 = 33.0 cm = 0.33 m

Width of the thread wall b_1 = 20 cm = 0.20 m

The moment of a uniform solid cylinder m and radius R is given by

$$I = \frac{1}{2}mR^2 = \frac{1}{2}(\rho V)R^2 = \frac{1}{2}(\rho\pi R^2 b)R^2 = \frac{1}{2}\rho\pi b R^4$$

Hence the moment of inertia of the tread wall will be given by



MI of the solid cylinder of radius R_1 - MI of the inner cylinder removed.

$$= \frac{1}{2}\rho\pi b R_1^4 - \frac{1}{2}\rho\pi b R^4 = \frac{1}{2}\rho\pi b\,(R_1^4 - R^4)$$

Hence the moment of inertia of the thread wall will be given by

$$I_2 = \frac{1}{2}\rho\pi b\left(R_1^4 - r_1^4\right) = 0.5*1.10*10^3*3.1416*0.20*\left(0.33^4 - 0.305^4\right)$$

Or $I_2 = 1.1077$ kg.m^2

Hence the moment of inertia of the tire about its symmetry axis is given by

$I = I_1 + I_2 = 0.1736 + 1.1077 = 1.2815$ kg m^2