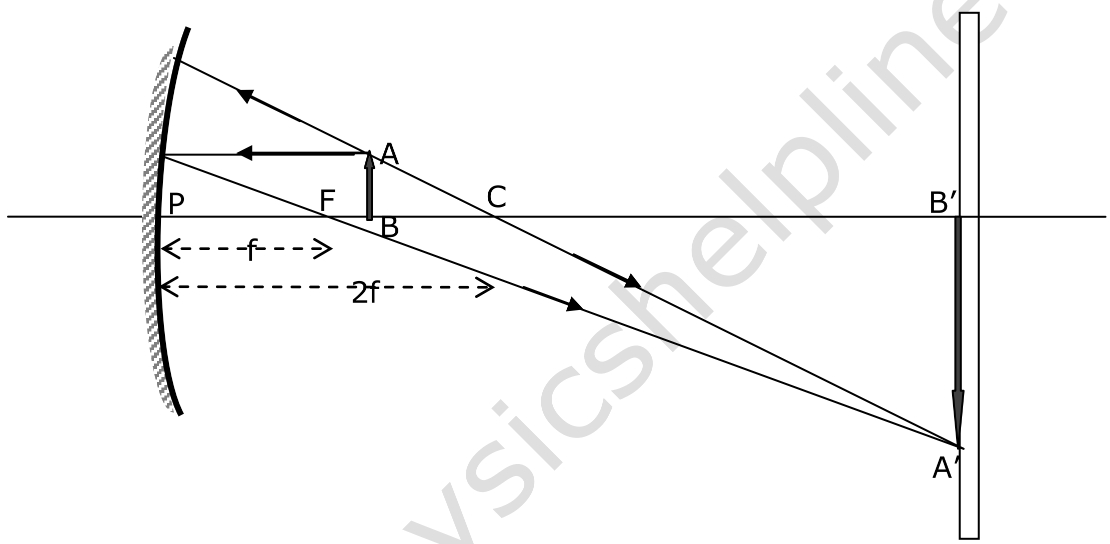
Q- An object is placed a distance r in front of a wall where r is exactly equals the radius of curvature of a certain concave mirror.
(a) At what distance from the wall the concave mirror to be placed so that a sharp image of the object is formed on the wall?
(b) What is the magnification of this image?

(a) To form image on the wall the image should be a real image and to get real image of an object using concave mirror, the object distance must be greater than the focal length of the mirror.

The ray diagram is shown bellow.



Let the distance of the mirror from the wall be d and this will be the image distance from the pole of the mirror.

As the object is placed at a distance **r = 2f** from the wall its distance from the mirror will be d – 2f.

Thus

The object distance $p = d - 2f$

Image distance $q = d$

Focal length of the mirror $f = f$

(All the distances measured in same direction, opposite to that of incident rays, thus positive)

Substituting in mirror formula we get

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

Or $$\frac{1}{f} = \frac{1}{d-2f} + \frac{1}{d}$$

Or $$\frac{1}{f} = \frac{d+d-2f}{(d-2f)*d}$$

Or $$d^2 - 2fd = 2fd - 2f^2$$

Or $$d^2 - 4fd + 2f^2 = 0$$

Or $$d = \frac{4f \pm\sqrt{16f^2-4*1*2f^2}}{2*1} = \frac{4f \pm 2\sqrt{2}\, f}{2}$$

Gives $d = \left(2 \pm \sqrt{2}\right)f$

As the distance between the wall and the mirror cannot be less then 2f the acceptable solution is

$$d = \left(2 + \sqrt{2}\right)f$$

As the radius of curvature r = 2f we get

$$d = \left(2 + \sqrt{2}\right) * \frac{r}{2}$$

Or $$d = \left(\mathbf{1} + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\right) * \boldsymbol{r}$$

(b) The magnification due to a concave mirror for real image is given by

$$m = \frac{I}{O} = \frac{q}{p}$$

Thus substituting object distance and image distance we get

$$m = \frac{d}{d-r} = \frac{\left(1+\frac{1}{\sqrt{2}}\right)*r}{\left(1+\frac{1}{\sqrt{2}}\right)*r-r} = \frac{1+\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \sqrt{2} + 1$$

(Sometimes it is shown with a negative sign which shows that the image is inverted)