Q- Two very long coaxial cylindrical conductors are placed with their axes coincide with z axis. The inner solid cylinder has radius a = 2 cm and caries a total current of I_1 = 1.2 A in the positive z-direction. The outer cylinder has an inner radius b = 4 cm, outer radius c = 6 cm and carries a current of I_2 = 2.4 A in the negative z-direction (pointing into the screen). The current is uniformly distributed over the cross-sectional area of the conductors. What is magnetic field at a distance of r = 5 cm from the z axis?

The magnetic field at a point due to a current distribution can be given by using Ampere's law.

According to this law the line integral of the magnetic field is equal to the current within the loop. Mathematically this can be written as

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 I$$

Where $\vec{B} \bullet d\vec{l}$ gives the product of the length (circumference) of the loop and the component of the magnetic field along the loop.

If the current is distribution is circularly uniform, the circular loop for which the magnetic field at all points equal in magnitude and parallel to it then $\oint \vec{B} \bullet d\vec{l}$ is given by B*2πR where R is the radius of the loop.

Now consider an imaginary loop passing through the given point P and its center coincides with the axis of the cylinders and whose plane is normal to the axis (red circle in diagram). The magnetic field at every point of this loop will be parallel to it and its magnitude will be constant, let B. As the loop length and the magnetic field are parallel the angle between them everywhere is zero and we have

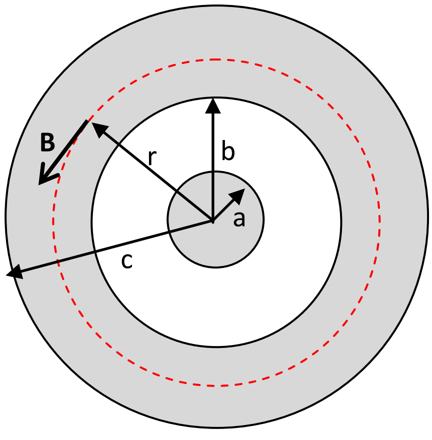
$$\oint \vec{B} \bullet d\vec{l} = \oint B.dl.\cos\theta = B\oint dl.1 = B.2\pi r = \mu_0 I_{in}$$

Where I_{in} is the current within the loop

Gives $B. = \dfrac{\mu_0 I_{in}}{2\pi r}$

[Hence using Ampere's law for any cylindrically symmetric current we can find the magnetic field at any point with knowing the current within the loop and the distance of the point from the axis of the current distribution.]

Now to our problem



Current through the inner cylinder I_1 = 1.2 A

Current through the outer cylinder I_2 = 2.4 A

Cross-section area of the outer cylinder $A = \pi (c^2 - b^2)$

Hence the current per unit area of the outer conductor is given by (current density)

$$j = \frac{I_2}{\pi\left(c^2 - b^2\right)}$$

Area of the outer conductor within the Amperean loop is A′ = π (r^2 - b^2) hence the current in outer conductor within the loop will be given by

$$I_2' = \frac{I_2\pi\left(r^2 - b^2\right)}{\pi\left(c^2 - b^2\right)}$$

As the current in the inner cylinder is along z direction and that the outer along negative z direction the net current within the loop of radius r is given by

$$I_n = I = I_1 - I_2' = I_1 - \frac{I_2\left(r^2 - b^2\right)}{\left(c^2 - b^2\right)}$$

And hence using Ampere's law the magnitude of the field B at P is given by

$$B. = \frac{\mu_0}{2\pi r}\left(I_1 - \frac{I_2\left(r^2 - b^2\right)}{\left(c^2 - b^2\right)}\right)$$

Or $$B. = \frac{2*10^{-7}}{5.0*10^{-2}}\left(1.2 - \frac{2.4\left(5^2 - 4^2\right)}{\left(6^2 - 4^2\right)}\right)$$

Or B = $4.8*10^{-7}$ T

[As all radii are in cm need not to convert for ratio of radii squared inside]