Q- A bungee jumper whose mass is 95 kg, has just completed his first bungee jump and is now bouncing up and down at the end of the cord. His oscillations have initial amplitude of 7 m and a period of 4.0 s.

(a) What is the spring constant of the bungee cord?

The time period of a simple harmonic motion is given by

$$T = 2\pi\sqrt{\frac{m}{K}}$$

Gives $K = \frac{4\pi^2 m}{T^2} = \frac{4*3.14^2*95}{16.0} = 234.16\,\text{N/m}$

(b) What is jumper's maximum speed while oscillating?

The total energy of oscillation is given by the maximum potential energy as

$$U = \tfrac{1}{2} KA^2 = 0.5*234.16*7^2 = 5736.92\ \text{J}$$

The speed is maximum when the whole potential energy of oscillation will convert in to kinetic energy and hence

$$\frac{1}{2} m v_{\max}^2 = 5736.92$$

Gives $v_{\max}^2 = \frac{5736.92*2}{m} = \frac{5736.92*2}{95} = 120.77$

Or $v_{\max} = \sqrt{120.78} = 10.99\,\text{m/s}$

(c) From what height above the lowest point did the jumper jump?

The extension for the equilibrium position is

$$x_0 = mg/K = 95*9.8/234.16 = 3.976\ \text{m}$$

Hence total extension in the cord for the lowest point will be

$$x_0 + A = 3.976 + 7 = 10.976\ \text{m}$$

At the lowest point the velocity and hence the kinetic energy is zero. Thus according to law of conservation of energy

loss in gravitational PE = gain in elastic potential energy

or $mgh = \tfrac{1}{2} K(x_0 + A)^2$

gives $h = 234.16*10.976^2/(2*95*9.8) = 15.15\ \text{m}$

(d) If the damping constant due to air resistance is 6.0 kg/s, how many oscillations will the jumper make before his amplitude has decreased to 4.0 m?

The amplitude as a function of time for a damped oscillation is given by

$$A(t) = A_0 e^{-(b/2m)t}$$

Where A_0 is the initial amplitude, b is damping constant and m is the mass of the object. Hence substituting the values we have

$$4.0 = 7.0 * e^{-(6.0/2*95)t}$$

Gives $e^{0.0316*t} = 7/4 = 1.75$

Or $0.0316*t = \ln 1.75$

Or $t = 0.56/0.0316 = 17.72$ s

Hence the number of oscillations in this time will be

$17.72/4.0 = 4.43$
