Q- (a) A positive ion of mass m and charge q, initially at rest is accelerated through a potential difference V. Find its speed in terms of given quantities.

The potential is the energy stored per unit charge hence the difference in the electrostatic potential energy of charge q_1 when the potential is change by V is given by qV

Thus the loss in electrostatic potential energy when the charge is accelerated through a potential difference V is qV.

As there is no non-conservative force is to be considered, the total energy of the system remains conserved and hence applying law of conservation of energy for the charged particle we get

Gain in kinetic energy = loss in electrostatic potential energy

Or $\frac{1}{2}mv^2 = qV$

Gives $v = \sqrt{\frac{2qV}{m}}$ ---------------------------- (1)

(b) The ion than enters a region of uniform magnetic field B, normal to its motion, and moves in a semicircular orbit of radius r. Calculate its radius in terms of given quantities.

The force acting on a charge q moving with velocity v in a magnetic field B is given by Lorentz formula as

$$\vec{F} = q(\vec{v} \times \vec{B})$$

Or $F = qvB\sin\theta$

Here θ is the angle between the direction of motion and the magnetic field. As the velocity and magnetic field are perpendicular to each other, $\theta = 90^0$ and $\sin\theta = 1$

Thus the force on the ion will be

$F = qvB$ ---------------------------- (2)

Now this force is always perpendicular to the direction of motion, will behave as the centripetal force and thus related to its velocity as

$F = \frac{mv^2}{r}$ ---------------------------- (3)

From equation (2) and (3) we get

$$F = \frac{mv^2}{r} = qvB$$

Or $\frac{mv}{r} = qB$

Gives $R = \frac{mv}{qB}$

Substituting for v we get

Or $R = \frac{1}{B}\sqrt{\frac{2mV}{q}}$

(c) What is the minimum magnitude of the electric field required in the region so that the ion moves in a straight line in the region?

If the electric field E is applied such that will apply a force qE on the charge equal to the force due to magnetic field and in the direction opposite to it, the resultant of the two forces will be zero and the ion will move in a straight line.

Thus for the particle to go straight

qE = q v B

Or E = v*B

Substituting value of v we get

$$E = B\sqrt{\frac{2qV}{m}}$$

Thus for the balancing of electrostatic and magnetic force the ratio of electric and magnetic fields must be equal to the speed of the ion.