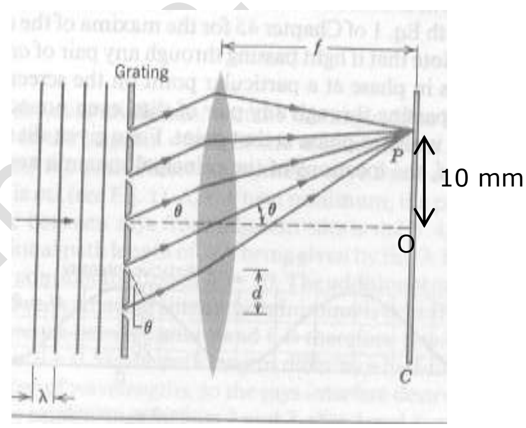
Q- A diffraction grating has 20 slits per mm. The distance between the slit centers is twice the width of each slit. The grating is uniformly illuminated at normal incidence. The transmitted light passes through a lens of focal length 1 m and then onto a screen that lies 1 m from the lens. On the screen the first-order diffraction peak lies 10mm from the centre-line of the apparatus. Calculate the wavelength of the light and predict the location of the next visible diffraction peak.

As light is considered a wave, every slit can be considered as a secondary source of light emitting waves in all directions. If the incidence of the parallel beam is normal, all these sources are coherent (same phase). The rays from consecutive slits going in a particular direction (say angle $\theta$ with the central line) will have same path difference.

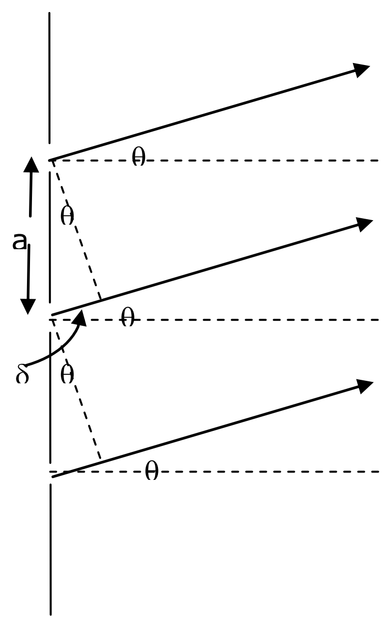
For a diffraction grating if a is the width of the slit and b is the width of the line (opaque) then for two nearest rays going in direction $\theta$ will have a path difference of $\delta = (a+b)\sin\theta$

All the rays going in the direction for which the path difference $\delta$ is integer multiple of wavelength $\lambda$, will be in same phase and interfere constructively when focused to a point on the screen and gives maximum intensity at that point.




The rays parallel to central line ($\theta = 0$) are already in same phase and giving maxima on the screen, called central maxima. As we move away on the screen form this point intensity decreases, reaches a minimum and it again increases to give maximum called first maxima. This will be the point where those rays are focused which is going in such a direction for which the nearest rays have a path difference $\delta = \lambda$ and so on.

The angle $\theta$ corresponding to the m$^{th}$ order maximum is given by




$$(a+b)\sin\theta_m = m\lambda$$

$$\Rightarrow \sin\theta_m = \frac{m\lambda}{(a+b)} = N * m\lambda$$

Where m is 0, 1, 2, …….. and N is the number of lines per unit length of the grating.

For the central maxima at O, m =0 gives $\theta = 0$

For the first order maxima at P, m = 1 gives

$\sin\theta = N\lambda$

but as the distance of the first maxima P form O is $x_1 = 10$ mm and the distance between the lens and the screen is f = 1m solving the right angled triangle we get

$$sin\theta = \frac{x_1}{\sqrt{f^2+x_1^2}}$$

Hence from above two equations we get

$$N\lambda = \frac{x_1}{\sqrt{f^2+x_1^2}}$$

Or $$\lambda = \frac{x_1}{N\sqrt{f^2+x_1^2}}$$

Now N = 20 per mm = 20000 per m

$x_1$ = 10 mm = 0.01 m

f = 1 m

Hence $\lambda = \frac{x_1}{N\sqrt{f^2+x_1^2}} = \frac{0.01}{20000\sqrt{1+0.01^2}} = 5 * 10^{-7} m = 500\ nm$

Hence the wavelength of the light used is **500 nm**.

For the second order maxima m = 2 gives

$$\sin\theta_2 = 2\ N\ \lambda$$

Thus we get

$$\frac{x_2}{\sqrt{f^2+x_2^2}} = 2N\lambda$$

Or $$\frac{x_2}{\sqrt{f^2+x_2^2}} = 2 * 20000 * 500 * 10^{-9} = 0.02\ m$$

Or $${x_2}^2 = (0.02)^2 * (f^2 + x_2^2)$$

Or $$x_2 = \frac{0.02*1}{0.9998} = .02\ m = 20.0\text{ mm}$$