

Q- A circular loop of radius $a = 20$ cm and $N = 168$ turns is fixed in the x - y plane. A spatially uniform magnetic field with only a z -component covers the entire area of the loop. The plot at the right shows B_z measured in tesla versus time t measured in seconds. The $+z$ direction is OUT of the screen.

(a) Calculate the *magnetic flux* through the loop in the z -direction at the times indicated below. In each case, specify the direction by giving a positive value for flux in the $+z$ direction and a negative value for flux in the $-z$ direction.

The flux through a loop of area A , in a magnetic field B and having number of turns N is given by

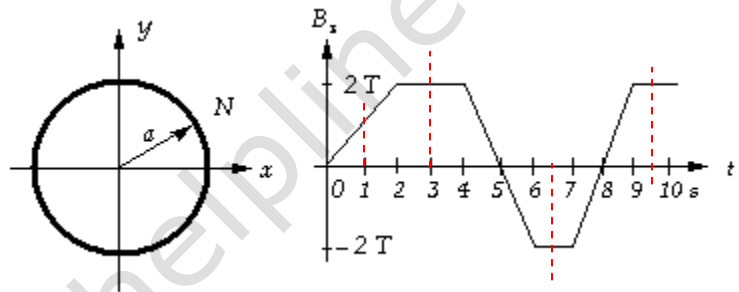
$$\phi = N \cdot B \cdot A \cos \theta$$

Where θ is the angle between area vector A and the field B .

Or $\phi = N \cdot B \cdot \pi a^2 \cos \theta$

Or $\phi = 168 \cdot B \cdot 3.14 \cdot 0.2^2 \cos \theta$

Or $\phi = 21.1 \cdot B \cos \theta$



As the positive direction of the field and area both are the same ($+z$), $\theta = 0$ and hence $\cos \theta = 1$ hence

$$\phi = 21.1 \cdot B$$

Substituting the values of the magnetic field from the graph we get

At $t = 1.0$ s the field $B = 1$ T gives

$$\phi_1 = 21.1 \cdot 1 = 21.1 \text{ T-m}^2$$

At $t = 3.0$ s the field $B = 2$ T gives

$$\phi_2 = 21.1 \cdot 2 = 42.2 \text{ T-m}^2$$

At $t = 5.0$ s the field $B = 0$ T gives

$$\phi_3 = 0 \text{ T-m}^2$$

At $t = 6.5$ s the field $B = -2$ T gives

$$\phi_4 = 21.1 \cdot (-2) = -42.2 \text{ T-m}^2$$

At $t = 8$ s the field $B = 0$ T gives

$$\phi_5 = 21.1 \cdot 0 = 0 \text{ T-m}^2$$

At $t = 9.5$ s the field $B = 2$ T gives

$$\phi_6 = 21.1 \times 2 = 42.2 \text{ T}\cdot\text{m}^2$$

(b) Calculate the *induced EMF* (electromotive force) in the loop at the times specified below. In each case specify the sense of the EMF by giving a positive value for a counterclockwise EMF as viewed in the above figure; specify a clockwise sense by giving a negative value.

The induced EMF is given by the faraday's law as

$$E = - d\phi/dt = - N \cdot A \cdot (dB/dt) = - 21.1 \cdot (dB/dt)$$

The rate of change of field is given by the slope of the graph

In first two second the field increases from 0 to 2T by 2T in 2 sec and hence the slope is $2/2 = 1$ T/s and hence the induced EMF will be

$$E_1 = - 21.1 \cdot 1 = -21.1 \text{ V}$$

The direction is in accordance to the Lenz law

At $t = 3.0$ s the graph is parallel to t axis means B is constant and hence $dB/dt = 0$ so induced EMF is also zero.

$$E_2 = 0$$

From 4 to 6 s the field changes uniformly from 2T to -2T in 2 sec hence $dB/dt = - 2$ T/s hence at $t = 5$ s

$$E_3 = -21.1 \cdot (-2) = +42.2 \text{ V}$$

At $t = 6.5$ s the graph is parallel to t axis means B is constant and hence $dB/dt = 0$ so induced EMF is also zero

$$E_4 = 0$$

From 7 to 9 s the field changes uniformly from -2T to 2T in 2 sec hence $dB/dt = 2$ T/s hence

$$E_5 = -21.1 \cdot 2 = - 42.2 \text{ V}$$

At $t = 9.5$ s the graph is parallel to t axis means B is constant and hence $dB/dt = 0$ so induced EMF is also zero

$$E_6 = 0$$