Q- Two charges placed on the x axis. $Q_a = 4 \ \mu\text{C}$ at x = 0 and $Q_b = -4 \ \mu\text{C}$ at x = 22cm.

(a) Find the net electric field at point P, at x = -6 cm.



The electric field at a point at distance r from a positive charge Q is given by

$$\vec{E} = \frac{Q}{4\pi \in_0 r^2} * \hat{r}$$

Here \hat{r} is the unit vector along the line joining the charge to the point.

Hence the field at P due to charge Qa will be

$$\vec{E}_a = \frac{Q_a}{4\pi \in_0 d^2} * (-\hat{i}) = \frac{9*10^9*4*10^{-6}}{0.06^2} * (-\hat{i}) = 1.0*10^7 (-\hat{i}) \text{ N/C}$$

Here \hat{i} is the unit vector in x direction and negative as the field is in negative x direction. Similarly the field due to Qb will be

$$\vec{E}_{b} = \frac{Q_{b}}{4\pi \epsilon_{0} (a+d)^{2}} * (-\hat{i}) = \frac{9*10^{9}*(-4*10^{-6})}{0.28^{2}} * (-\hat{i}) = 0.0459*10^{7}\hat{i}$$

According to the law of superposition of the field the resultant field is given by the vector sum of the two fields and hence the resultant field at P is given by

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = -1.0 * 10^7 \,\hat{\imath} + \ 0.459 * 10^7 \,\hat{\imath} = \ -0.541 * 10^7 \,\hat{\imath}$$

Hence the magnitude of the net electric field at P is **0.541*10⁷ N/c** and its direction is along negative x direction.

(b) Find the force on Q_b due to Q_a .

The force on Qb due to Qa is given by

$$\vec{F}_{ba} = \frac{Q_a Q_b}{4\pi \in_0 r_{ab}^2} \hat{r}_{ab}$$

Where \hat{r}_{ab} is the unit vector from Qa to Qb.

Or
$$\vec{F}_{ba} = \frac{9*10^9 (4*10^{-6})(-4*10^{-6})}{(0.22)^2} \hat{i} = -2.975 \hat{i} \text{ N}$$

Means the magnitude of the force is 2.975 N and it is in negative x direction.

(c) The charges Q_a and Q_b are now attached to the ends of a spring whose un-stretched length is $s_0 = 22$ cm. With the charges attached, the spring compresses to an equilibrium length $s_1 = 12$ cm. Calculate the spring constant k_s of the spring.

The spring constant of the spring is the force required per unit extension and hence if the change in length of the spring is ΔI when force F is applied to the spring,, the force constant is given by

$$K = F/\Delta I$$

Now the change in the length of the spring is $\Delta I = 22 - 12 = 10$ cm = 0.1 m

And the force of attraction between the charges is

$$F = \frac{Q_a Q_b}{4\pi \in_0 l^2} = \frac{9*10^9 \left(4*10^{-6}\right) \left(-4*10^{-6}\right)}{\left(0.12\right)^2} = 10N$$

Hence the force constant of the spring is given by $K = F/\Delta I = 10/0.1 = 100 \text{ N/m}$

 $k_s = 100.0 \text{ N/m}$