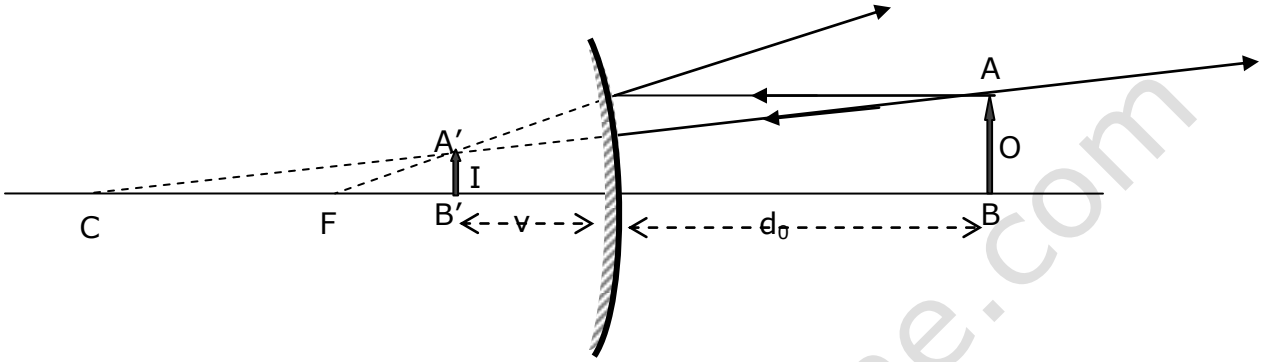
Q- if the focal length of a convex mirror be written as -|f|, show that the magnification m of an object at a distance d0 from this mirror is given by m = |f|/(d0+|f|).Based on this relation , explain why your nose looks bigger than the rest of your face when looking into a convex mirror.



Let the object AB of length O is placed at a distance d_0 from the pole of mirror and its image A'B' is formed at distance v behind the mirror as shown by the ray diagram. Here

The object distance p = + $|d_0|$
The image distance q = - $|d|$
And the focal length of the mirror f = - $|f|$

As the image distance q, object distance p and the focal length f of the mirror are related as (mirror formula)

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

Substituting the values with proper sign we get

$$\frac{1}{-|f|} = \frac{1}{|d_0|} + \frac{1}{-|d|}$$

Gives $|d| = \frac{|d_0|*|f|}{|d_0|+|f|}$ ---------- (1)

The magnification of image is the ratio of image length to the object length and thus we get

$$m = \frac{I}{O} = \frac{A'B'}{AB}$$ ---------- (2)

Now as we know that for a spherical mirror the radius curvature is twice of the focal length, the distance between the center of curvature and the image I will be

CB' = 2f – d

And the distance between the object and the center of curvature will be

CB = 2f + d_0

Considering the right angled triangles CA'B' and CAB, as they are similar triangles, the ratios of corresponding sides are equal and hence we get

$$\frac{A'B'}{AB} = \frac{CB'}{CB}$$

Substituting value of the sides from above we get

$$m = \frac{2|f| - |d|}{2|f| + |d_0|}$$

Substituting value of d from equation (1) we get

$$m = \frac{2|f| - \frac{|d_0|*|f|}{|d_0|+|f|}}{2|f| + |d_0|}$$

Or $$m = \frac{2|f|*(|d_0|+|f|) - |d_0|*|f|}{(2|f| + |d_0|)*(|d_0|+|f|)}$$

Or $$m = \frac{2|f|*|d_0| + 2|f|^2 - |d_0|*|f|}{(2|f| + |d_0|)*(|d_0|+|f|)}$$

Or $$m = \frac{|f|*|d_0| + 2|f|^2}{(2|f| + |d_0|)*(|d_0|+|f|)} = \frac{|f|*(|d_0| + 2|f|)}{(2|f| + |d_0|)*(|d_0|+|f|)}$$

Or $$m = \frac{|f|}{(|d_0|+|f|)}$$

Hence shown

As in above expression the denominator will always be greater than the numerator, the magnification will be less than one and thus the image will be diminished.

Further as we move away from the central axis because of the curvature of the mirror the distance of the object from the mirror surface increases and hence with increase in d_0 the magnification further decreases. Hence the image of the points near the central axis (paraxial) will be larger as compared to the points away from the central axis. This is why the image of nose is larger than the other parts.