Q- Big ben the parliament tower clock in London, has hour and minute hands with lengths of 2.70 m and 4.50m and masses of 60.0kg and 100kg respectively. Calculate the total angular momentum of these hands about the center point. Treat the hands as long thin uniform rods.

The angular velocity of a body about a point is the angle turned by the body in unit time or it is defined as the angular displacement per unit time and is denoted by θ.

When a body turns by a complete rotation about the axis of rotation, its angular displacement is $\theta = 2\pi$ radians.

The time in which a body makes one complete rotation is called its time period and denoted by T.
Hence the angular velocity ω of a body is given by

$$\omega = \frac{2\pi}{T}$$

The hour hand makes one complete rotation in 12 hours = 43200 s hence its angular velocity will be

$$\omega_H = \frac{2\pi}{T_H} = \frac{2\pi}{43200} = 1.4544 * 10^{-4} \text{ Radians/sec.} \quad \text{---------- (1)}$$

The min hand makes one complete rotation in 1 hour = 3600 s hence its angular velocity will be

$$\omega_M = \frac{2\pi}{T_M} = \frac{2\pi}{3600} = 1.7453 * 10^{-3} \text{ Radians/sec.} \quad \text{---------- (2)}$$

Now moment of inertia of a thin uniform rod of mass M and length L about one of its ends is given by

$$I = \frac{1}{3} ML^2$$

Hence the moment of inertia of the hour hand will be

$$I_H = \frac{1}{3} M_H {L_H}^2 = \frac{2.70 * (60.0)^2}{3} = 3240 \text{ Kg.m}^2. \quad \text{---------- (3)}$$

And the moment of inertia of the minute hand will be

$$I_M = \frac{1}{3} M_M {L_M}^2 = \frac{4.50 * (100.0)^2}{3} = 15000 \text{ Kg.m}^2. \quad \text{---------- (4)}$$

The angular momentum of a body is the quantity of the rotational motion of the body or the motional effect of the rotating body and is given by the product of its moment of inertia about the axis of rotation and angular velocity, and its magnitude is given by

$$J = I * \omega$$

Hence the angular momentum of the hour hand of the clock is

$$J_H = I_H \omega_H = 3240 * 1.4544 * 10^{-4} = 0.4712 \text{ Kg m}^2/\text{s}$$

And the angular momentum of the minute hand of the clock is

$$J_M = I_M \omega_M = 15000 * 1.7453 * 10^{-3} = 26.18 \text{ kg m}^2/\text{s}$$

Thus the total angular momentum of the two hands will be

$$J_H + J_M = 0.4712 + 26.18 = 26.65 \text{ Kg m}^2/\text{s}$$