Q- A circular loop with radius a = 0.35 m and N = 21 turns lies in the plane of the page. The wire used in constructing the loop has a resistance per unit length of dR/dl = 0.11 Ω/m. A spatially uniform magnetic field points into the page. In the interval between t = 0 and 10 s, the strength of this field varies according to the expression $B(t)$ = 0.007 t^3 T/s^3.

(a) Calculate the magnitude of the induced EMF in the coil at t = 5 s.

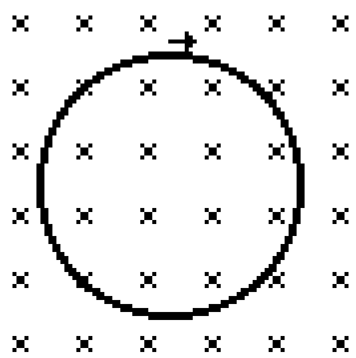
Area of the loop A = $\pi a^2 = 3.14*(0.35)^2 = 0.385\ m^2$

If the field at a time t is B, then the flux through the loop will be given by

$$\phi_B = NBA\cos\theta = -NBA$$ [As the field is in -z direction]

Hence the induced EMF is given by

$$E = -\frac{d\phi_B}{dt} = NA\frac{dB}{dt}$$ ---------------- (1)



Now the magnetic field is B = $0.007*t^3$

Hence $\frac{dB}{dt} = 0.007*3t^2 = 0.021*t^2$

And hence the induced EMF at time t is given by

E = $21*0.385*0.021t^2 = 0.17*t^2$

And thus the induced EMF at t = 5 s will be

E = 0.17*25 = 4.24 V

(b) Calculate the current in the windings at t = 5 s.

Length of the wire in the loop is given by

l = $2\pi a*N$ = 2*3.14*0.35*21 = 46.16 m

Hence the resistance of the loop will be

R = 46.16*0.11 = 5.08 Ω

And hence the current in the loop will be

I = E/R = 4.24/5.08 = 0.835 A

As the flux in negative z direction and increasing, according to Lenz law the induced current will be in such a direction that it will try to decrease the flux in that direction or the flux through the coil and hence field due to induced current will be in +z direction. Hence using right hand rule we can say that the current will be in counter clock wise direction.

(c) In the time interval between 0 and 10 s, how much electrical charge passes any given point in the windings? (Give magnitude only.)

According to faraday's law of electromagnetic induction

$$E = -\frac{d\phi}{dt}$$

If the resistance of the loop is R then the current in the loop I = dq/dt is given by

$$I = \frac{dq}{dt} = \frac{E}{R} = -\frac{1}{R}\frac{d\phi}{dt}$$

Gives $dq = -\frac{1}{R}d\phi$

Hence **magnitude** of the charge flowing through the loop is given by

$$\Delta q = \frac{\Delta\phi}{R} = \frac{N * A * \Delta B}{R}$$

Now in time t = 0 to 10 s the field increases from 0 to $0.007*10^3$ T

Hence we have

$$\Delta q = \frac{21 * 0.385 * 7}{5.08} = 11.14.C$$