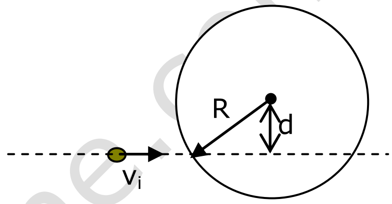
Q- A wad of sticky clay with mass m and velocity vi is fired at a solid cylinder of mass M and radius R. The cylinder is initially at rest and is mounted on a fixed horizontal axle that runs through its center of mass. The line of motion of the projectile is perpendicular to the axle and at a distance d<R from the center. Find the angular speed of the system just after the clay strikes and sticks to the surface of the cylinder.

The problem is based on the law of conservation of angular momentum. According to this law if there is no external torque on the system, its angular momentum remains conserved.

Now the angular momentum of a translating body about an axis is the moment of its momentum about the axis of rotation. This means the product of momentum of the body and the perpendicular distance of the lime of its motion from the axis of rotation.

Initial Momentum of the wad just before impact

$P = m*v_i$



Distance of lime of motion from the axis of the cylinder = d

Hence angular momentum of the wad just before impact about the axis of the cylinder

$J = P*d = m*v_i*d$

Angular velocity and hence the angular momentum of the cylinder before impact is zero

Thus total angular momentum of the wad plus cylinder system before impact will be

$J_I = m*v_i*d$

Moment of inertia of the cylinder about its axis is (½) MR^2

Moment of inertia of the wed after it sticks to the surface of the cylinder is mR^2

Hence moment of inertia of the clay cylinder system after impact will be

$$I = \frac{1}{2}MR^2 + mR^2$$

If the angular velocity of the clay cylinder system after impact will be □ then its angular momentum after impact will be

$$J_F = I\omega = \left(\frac{1}{2}MR^2 + mR^2\right)\omega$$

Now as there is no external torque about the axis of the cylinder during the short time of impact, the angular momentum of the system about the axis remains conserved and hence we have

$$J_F = J_I$$

Or $$\left(\frac{1}{2}MR^2 + mR^2\right)\omega = mv_i d$$

Gives $$\omega = \frac{mv_i d}{\left(\frac{1}{2}MR^2 + mR^2\right)} = \frac{2mv_i d}{(M + 2m)R^2}$$