Q- A long straight wire carrying a current of $I_{1}=30 \mathrm{~A}$ is paced in the plane of a rectangular wire loop and parallel to its nearest side of length $I=30 \mathrm{~cm}$, at a distance of $a=1 \mathrm{~cm}$. The width of the loop is $\mathrm{b}=8 \mathrm{~cm}$. Find the magnitude of the net force on the loop if the current in the loop is $I_{2}=20 \mathrm{~A}$.

The magnetic field due to a long thin wire carrying current I at a distance $r$ from it is given by

$$
B=\frac{\mu_{0} I}{2 \pi r}
$$

As the current in the long wire is upward according to Ampere's right hand rule the magnetic field on the right of the wire is into the page. Thus the current in the loop is perpendicular everywhere to the field direction and hence the magnitude of the force on the wire is given by

$$
F=B I l
$$

Now the field at a distance of $a=1 \mathrm{~cm}=0.01 \mathrm{~m}$ from the long
 wire (at the nearer parallel wire) will be

$$
B_{1}=\frac{\mu_{0} I_{1}}{2 \pi a}
$$

Thus the magnitude of the force on the wire of the loop nearest to the long wire is given by

$$
\begin{equation*}
F_{1}=B_{1} * I_{2} * l=\frac{\mu_{0} I_{1} I_{2} l}{2 \pi a} \tag{1}
\end{equation*}
$$

The current in the nearest side of the loop is also upward thus according to Flemings left hand rule the direction of the force on it will be towards left.

Now the field at a distance of $b=(1+8) \mathrm{cm}=0.09 \mathrm{~m}$ from the long wire (at the farther parallel wire) will be

$$
B_{2}=\frac{\mu_{0} I_{1}}{2 \pi b}
$$

Thus the magnitude of the force on the wire of the loop farther to the long wire is given by

$$
\begin{equation*}
F_{2}=B_{2} * I_{2} * l=\frac{\mu_{0} I_{1} I_{2} l}{2 \pi b} \tag{2}
\end{equation*}
$$

The current in the farthest side of the loop is downward thus according to Fleming's left hand rule the direction of the force on it will be towards right.

The currents in the top and bottom sides off the loop are in opposite direction and thus the force on them are upward and downward directions. As both sides are symmetrically placed from the long wire, the two forces are equal in magnitude and thus the resultant of the force acting on the two sides will be zero.

Thus net force acting on the loop will be the resultant of $F_{1}$ and $F_{2}$. As the two force are in opposite directions the resultant is given by

$$
F=F_{1}+F_{2}=\frac{\mu_{0} I_{1} I_{2} l}{2 \pi a}-\frac{\mu_{0} I_{1} I_{2} l}{2 \pi b}=\frac{\mu_{0} I_{1} I_{2} l}{2 \pi}\left(\frac{1}{a}-\frac{1}{b}\right)
$$

Substituting the numerical values of the quantities we get

$$
F=\frac{4 \pi * 10^{-7} * 30 * 20 * 0.30}{2 \pi}\left(\frac{1}{0.01}-\frac{1}{0.09}\right)
$$

Or $\quad F=3.6 * 10^{-5}(100-11.11)$
Or $\quad F=3.2 * 10^{-3} \mathrm{~N}$
As the greater force is on the nearer wire towards left the net force will be to the left means towards the long wire.

