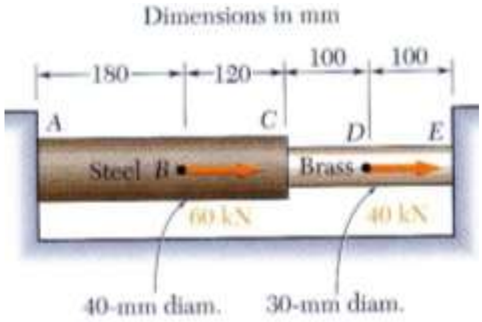
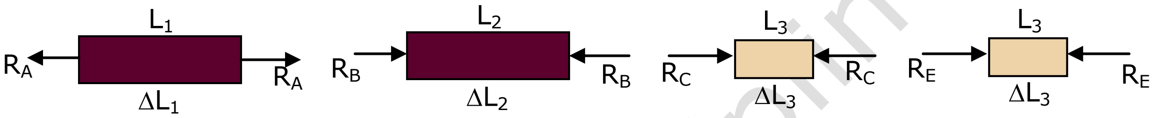
Q- Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained (fixed) by rigid supports at A and E with their natural lengths as shown in the figure. Now forces of 60 kN and 40 kN are applied at points shown. Knowing that E(steel)=200 GPa and E(brass)=105 GPa, determine
(a) the reaction at A and E
(b) the deflection of point C.

The forces applied produces the stress in the different parts of the system differently and these parts can be considered separately as parts AB, BC, CD and DE.



The external forces in each part produce strain and hence give rise to compressive of tensile stress accordingly.

(a) The reactions and the corresponding change in lengths of different parts are shown in the diagrams bellow.



Considering the first part

Tensile stress = $\dfrac{R_A}{\pi r_1^2}$ and strain = $\dfrac{\Delta L_1}{L_1}$

Hence according to hooks law stress = E*strain we have

$$\frac{R_A}{\pi r_1^2} = E * \frac{\Delta L_1}{L_1}$$ [r_1 and r_2 are radii of the two rods]

Gives $$\Delta L_1 = \frac{R_A * L_1}{\pi r_1^2 E_S} = \frac{180*10^{-3} R_A}{3.14*0.02^2*200*10^9} = 7.166*10^{-10}*R_A$$

Similarly for the other three part the change in their lengths can be given by

$$\Delta L_2 = -\frac{R_B L_2}{\pi r_1^2 E_S} = -\frac{R_B 120*10^{-3}}{3.14*0.02^2*2*10^{11}} = -4.777*10^{-10} R_B :$$

$$\Delta L_3 = -\frac{R_C L_3}{\pi r_2^2 E_B} = -\frac{R_C *100*10^{-3}}{3.14*0.015^2*1.05*10^{11}} = -1.348*10^{-9} R_C :$$

$$\Delta L_4 = -\frac{R_E L_4}{\pi r_2^2 E_B} = -\frac{R_E *100*10^{-3}}{3.14*0.015^2*1.05*10^{11}} = -1.348*10^{-9} R_E$$

The negative signs are due to compressions.

Now as we know that the total length of the system does not change we have

$$\Delta L_1 + \Delta L_2 + \Delta L_3 + \Delta L_4 = 0$$

Gives $7.166*10^{-10} R_A - 4.777*10^{-10} R_B - 1.348*10^{-9} R_C - 1.348*10^{-9} R_E = 0$ --- (1)

Now as we know that the net force at B is 60KN hence

$R_A + R_B = 60$ kN

Or $R_B = 60 \text{ kN} - R_A$ -------------- (2)

As there is no external force at C

$- R_B + R_C = 0$

or $R_C = R_B = 60 \text{ kN} - R_A$ -------------- (3)

And for point D

$- R_C + R_E = 40$ kN

Or $- (60 \text{ kN} - R_A) + R_E = 40$ kN

Or $R_E = 100 \text{ kN} - R_A$ -------------- (4)

Substituting the values of R_B, R_C and R_E in terms of R_A in equation (1) we have

$$7.166*10^{-10} R_A - 4.777*10^{-10}*(60 \text{ kN} - R_A) - 1.348*10^{-9}*(60 \text{ kN} - R_A) - 1.348*10^{-9}*(100 \text{ kN} - R_A) = 0$$

Or $(7.166*10^{-10} + 4.777*10^{-10} + 1.348*10^{-9} + 1.348*10^{-9}) R_A = 4.777*10^{-10}*60000 + 1.348*10^{-9}*60000 + 1.348*10^{-9}*100000$

Or $38.903 R_A = 2443420$

Or $\mathbf{R_A = 62808 \text{ N}}$

And hence

$\mathbf{R_E = 100000 - R_A = 37192 \text{ N}}$

(b) Substituting the values of R_A in equations 2 and 3 we get

$R_B = R_C = 60000 - 62808 = - 2808$ N

This negative sign show that the directions of forces are opposite to that we have taken means the first three parts get elongated and only last part is compressed.

The change in lengths of different parts is given by

$$\Delta L_1 = 7.166*10^{-10} * R_A = 7.166*10^{-10} * 62808 = 4.5*10^{-5} m$$
$$\Delta L_2 = -4.777*10^{-10}(-2808) = 1.34*10^{-6} m$$
$$\Delta L_3 = -1.348*10^{-9}(-2808) = 3.78*10^{-6} m:$$
$$\Delta L_4 = -1.348*10^{-9} * 37192 = -5.01*10^{-5} m$$

The last one is negative means compression.

Hence the displacement of point C will be

$x_C = \Delta L_1 + \Delta L_2 = (4.5 + 0.134)*10^{-5} = \mathbf{4.634*10^{-5} \text{ m}}$