Q- An electromagnetic wave has a frequency of 110 MHz and is traveling in a vacuum. The magnetic field is given by $\vec{\mathbf{B}}(z, t) = (2.00 \times 10^{-8}\ \mathrm{T}) \cos(kz - \omega t)\,\hat{\mathbf{i}}$.

(a) Find the wavelength and the direction of propagation of this wave.

The velocity of the electromagnetic waves is equal to the velocity of light c = $3*10^8$ m/s.

The frequency n, wavelength λ and the velocity c of a wave is related by c = n λ, hence the wavelength of the wave is given by

$$\lambda = c/n = 3*10^8/(110*10^6) = 2.73\ \mathrm{m}$$

The phase angle of the wave at any instant of time is decreasing in the direction of wave motion. Here with increase in z the phase angle increases and thus we can say that the wave is traveling in negative z direction.

(b) Find the electric field vector $\vec{\mathbf{E}}(z, t)$.

The electric and magnetic fields are in phase and are related as E = c*B we get the amplitude of electric field as

$$E_m = c*B_m = (3*10^8)*(2.00*10^{-8}) = 6.00\ \mathrm{V/m}$$

The wave number k is given by $k = 2\pi/\lambda = 2*3.1415/2.73 = 2.30\ \mathrm{m}^{-1}$

And the angular frequency is given by $\omega = 2\pi n = 2*3.1415*110*10^6$

Or $\quad \omega = 0.691*10^9\ \mathrm{s}^{-1} = 0.691\ \mathrm{ns}^{-1}$

Hence the electric field vector which is perpendicular to the magnetic field and direction of wave motion is given by

$$\vec{\mathbf{E}}(z, t) = (6.00\ \mathrm{V/m}) \cos(2.30\,z - 0.691\,t).\ \hat{\mathbf{j}}$$

(c) Determine the Poynting vector.

The Poynting vector is given by

$$\vec{\mathbf{S}}(z, t) = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right)$$

Or $$\vec{\mathbf{S}}(z, t) = \frac{1}{\mu_0} 6.00*2*10^{-8} \cos^2(2.30z - 0.691t)\left(\hat{j} \times \hat{i}\right)$$

Or $$\vec{\mathbf{S}}(z, t) = \frac{10^7}{4\pi} * 6.00*2*10^{-8} \cos^2(2.30z - 0.691t)\left(\hat{j} \times \hat{i}\right)$$

Or $$\vec{\mathbf{S}}(z, t) = 0.0955\cos^2(2.30z - 0.691t)\left(-\hat{k}\right)$$

(d) Using this Poynting vector, find the intensity of the wave.

The time average of magnitude of Poynting vector S gives the intensity I of the wave. As the time average of $\cos^2(2.30\,z - 0.691\,t)$ will be 1/2, the intensity of the wave is given by

$$I = 0.0955/2\ \mathrm{W/m^2} = 47.75\ \mathrm{mW/m^2}$$