Q- A particle is projected from the surface of the earth with a speed twice of the escape speed. Neglecting air resistance, find its speed when it is far away from the earth.

When the body is projected from the surface of earth and that too with the speed more than the escape speed, will definitely go to the outer space far from the earth's gravitational effect and still some energy will remain in it in form of kinetic energy.

As we know that the gravitational potential energy of an object of mass m at distance r from the center of earth is given by

$$U = -\frac{GMm}{r}$$

Thus the potential energy of the body at the surface of the earth will be

$$U = -\frac{GMm}{R}$$

Here M is the mass of the earth and R is its radius.

The escape velocity is the minimum velocity with which the body is projected and will go to the outer space. Thus just to send the body to outer space the energy to be given in form of kinetic energy to make net energy of the body equal to zero at the surface of earth or

$$\frac{1}{2}mv_{esc}^2 - \frac{GMm}{R} = 0$$

Or $\frac{1}{2}mv_{esc}^2 = \frac{GMm}{R}$ -------------------------------- (1)

Now if the body is projected with velocity double of escape speed then according to law of conservation of energy the kinetic energy of the body in the outer space will be equal to the net energy of the body at the surface of the earth and hence if the velocity of the body far away is v we get

$$\frac{1}{2}m(2 * v_{esc})^2 - \frac{GMm}{R} = \frac{1}{2}\, mv^2$$

Substituting value of GMm/R from equation (1) we get

$$\frac{1}{2}m(2 * v_{esc})^2 - \frac{1}{2}mv_{esc}^2 = \frac{1}{2}\, mv^2$$

Or $(2 * v_{esc})^2 - v_{esc}^2 = v^2$

Or $3v_{esc}^2 = v^2$

Gives $v = \sqrt{3}\, v_{esc}$

Thus the velocity of the body in outer space will be cube root three times the escape velocity.

(As escape speed in nearly 11.2 km/s the speed will be nearly 19.4 km/s)