

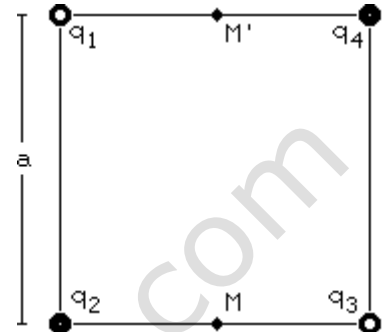
Q- Four charges  $q_1 = q_3 = -q$  and  $q_2 = q_4 = +q$ , where  $q = 7 \mu\text{C}$ , are fixed at the corners of a square with sides  $a = 2.2 \text{ m}$ .

(a) Calculate the x- and y-components of the net electric field at the midpoint  $M$  of the bottom side of the square.

The charges  $q_1$  and  $q_4$  constitutes a dipole of dipole moment  $P = q \cdot a$  and  $M$  is on the perpendicular bisector of it, hence the magnitude of the field at  $M$  due to this dipole will be given by the formula

$$E_1 = \frac{1}{4\pi \epsilon_0} \cdot \frac{qa}{\left[ a^2 + (a/2)^2 \right]^{3/2}} = \frac{8 \cdot q}{4\pi \epsilon_0 a^2 \cdot 5\sqrt{5}}$$

Or 
$$E_1 = \frac{(9 \cdot 10^9) \cdot 8(7 \cdot 10^{-6})}{(2.2)^2 \cdot 5\sqrt{5}} = 9.314 \cdot 10^3 \text{ N/C}$$



This field is in negative x direction.

The field due to  $q_2$  and  $q_3$  will be equal in magnitude and both are in positive x direction and their resultant will be

$$E_2 = 2 \cdot \frac{q}{4\pi \epsilon_0 (a/2)^2} = \frac{8q}{4\pi \epsilon_0 a^2} = \frac{(9 \cdot 10^9) \cdot 8(7 \cdot 10^{-6})}{(2.2)^2} = 1.04 \cdot 10^5 \text{ N/C}$$

This field will be in positive x direction.

As fields  $E_1$  and  $E_2$  both are in x direction the x component of the total field at  $M$  will be

$$104.1 \cdot 10^3 - 9.314 \cdot 10^3 = 94.79 \cdot 10^3 \text{ N/C}$$

And y component of the field at  $M$  will be zero.

(b) Find the total force exerted on  $q_4$  by the charges  $q_1$ ,  $q_2$ , and  $q_3$ :

Force on  $q_4$  due to  $q_1$  will be

$$\vec{F}_1 = \frac{q^2}{4\pi \epsilon_0 a^2} (-\hat{i})$$

Force on  $q_4$  due to  $q_2$  will be

$$\vec{F}_2 = \frac{q^2}{4\pi \epsilon_0 2a^2} (\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j}) = \frac{q^2}{4\pi \epsilon_0 2\sqrt{2} \cdot a^2} (\hat{i} + \hat{j})$$

Force on  $q_4$  due to  $q_3$  will be

$$\vec{F}_3 = \frac{q^2}{4\pi \epsilon_0 a^2} (-\hat{j})$$

Hence the total force will be

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \frac{q^2}{4\pi \epsilon_0 a^2} \left( \frac{1}{2\sqrt{2}} - 1 \right) (\hat{i} + \hat{j})$$

Or 
$$\vec{F} = \frac{(9 \cdot 10^9)(7 \cdot 10^{-6})^2}{2.2^2} (-0.6464) (\hat{i} + \hat{j}) = -0.0589 (\hat{i} + \hat{j})$$

Hence the magnitude of the force  $F = 0.0589 \cdot \sqrt{2} = \text{N}$  and its direction will bisect the angle between negative x and negative y axes.