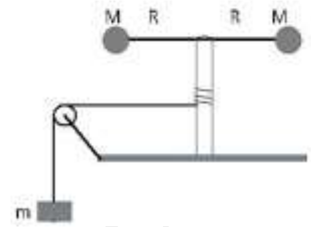
Q- A light cylindrical axle of radius r is free to rotate about its vertical axis. The upper end of the axle is attached to a light horizontal cross bar with two small masses, each M, at a distance R from the axle. A light string is wrapped on the axle and its free end is passing over a smooth light pulley and mass m is hanging through it. Find the ratio of tensions in the string and the cross bar as a function of time after the mass is released.



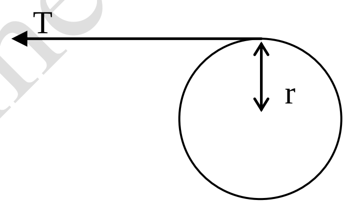
The string is wrapped on the axle and hence creating the torque to rotate the axle and the weights on the cross bar.

Let the tension in the string be T. writing the equation of motion for the hanging mass m we have its acceleration as

$mg - T = ma$ or $a = g - (T/m)$ ----- (1)

The product of tension T and the perpendicular distance from the axis of rotation, which is equal to the radius of the axle say r, give the torque acting on the axle hence

$\tau = T * r = I*\alpha$



Axle Top view

Here α is the angular acceleration of the axle. Hence

$$T = \frac{I\alpha}{r} = \frac{I*(a/r)}{r} = \frac{I*a}{r^2}$$

{$a = \alpha*r$ is the acceleration of the string}

Now considering the axle and the cross bar having negligible mass the moment of inertia of the weights (mass M each) is given by

$I = 2*MR^2$ (where R is the distance of the each weight from the center of axle.)

Hence substituting the value of I in the equation above we have

$$T = \frac{2MR^2 * \alpha * r}{r^2} = \frac{2MR^2 * \alpha}{r} \quad \text{--- (2)}$$

Now the angular velocity of the weights depends on the time t and is given by

$\omega = \alpha * t$

And the centripetal force required is given by the tension T' in the cross bar and hence we have $T' = M\omega^2 R = M\,\alpha^2 t^2 R$

Substituting the value of α from equation 2 in this equation we have

$$T' = MRt^2\left(\frac{T*r}{2MR^2}\right)^2 = \frac{T^2 r^2 t^2}{4MR^3}$$

This is the relation between the tension in the cross bar T' and that in the string with the hanging mass T.

Tension in the string is the constant and that in the cross bar will increase with the time.

Initially the tension in the cross bar is zero (not considering the vertical force exerted to hold the weight only longitudinal tension) and increases with the time. At some time it will be equal to T and afterwards will be greater than T.