Q- A light balloon is filled with $400m^3$ of helium at zero degrees Celsius. What is the maximum mass of the payload the balloon can lift?

The problem is based on Archimedes' Principle which states that "When a body is fully or partially immersed in a fluid there is an apparent loss in its weight. The apparent loss in the weight of the body is equal to the weight of the displaced fluid (equal to the volume of the body in the fluid)". This loss in the wait is due to the up-thrust (upward force) of the fluid.

The mass of the helium gas can be given by the product of its volume and density and hence

Mass of the helium (balloon) $m_{He} = V*\rho_{He}$

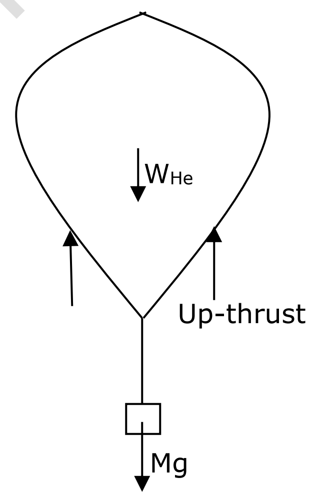
Thus, the weight of the balloon $W_{He} = m_{He}*g = V*\rho_{He}*g$

Here g is the acceleration due to gravity

As the balloon will displaced the same volume of air hence the up-thrust of the air is given by Weight of the air of the same volume V and hence

Weight of the displaced air $W_{Air} = m_{Air}*g = V*\rho_{Air}*g$

Since the density of air is more than that of Helium the weight of the air displaced or up-thrust will be grater then the weight of the balloon and hence the balloon can lift a pay lode such that the net force on it will be zero. If the maximum pay load without the descent of balloon has a mass M then in equilibrium condition



Net downward force = upward force

$W_{He} + Mg = W_{Air}$

Or $Mg = W_{Air} - W_{He}$

Or $Mg = V*\rho_{Air}*g - V*\rho_{He}*g = V*(\rho_{Air} - \rho_{He})*g$

Gives $M = V*(\rho_{Air} - \rho_{He})$ ------------------------- (1)

Now the volume of the balloon $V = 400\ m^3$

Density of air (from tables, Google) $\rho_{Air} = 1.2922$ gm/L $= 1.2922$ kg/m^3

Density of helium $\rho_{He} = 0.1786$ gm/L $= 0.1786$ kg/m^3

Plugging in the values in (1) the mass of the pay load can be given as

$M = 400*(1.2922 - 0.1786) = 400*1.1136 = 445.44$ kg.