Q- Two particles 1 and 2 move with constant velocities $\mathbf{v_1}$ and $\mathbf{v_2}$. At the initial moment, their radius vectors are $\mathbf{r_1}$ and $\mathbf{r_2}$. How must these four vectors be interrelated for the particles to collide?

For collision of two particles to collide, they must share the same position at the same instant of time.

As they are at different positions initially, there relative velocities must be in the direction of their relative position. If this is so they will collide with each other at some instant of time.

For this to happen direction of velocity of 2 relative to velocity of 1 must be towards the position of 1 relative to position of 2. Thus, taking unit vectors for directions we get

$$\frac{\vec{v}_2-\vec{v}_1}{|\vec{v}_2-\vec{v}_1|}=\frac{\vec{r}_1-\vec{r}_2}{|\vec{r}_1-\vec{r}_2|}$$

This is the required relation.

Q- A ship moving along the equator to the east with velocity v_0 = 30 km/hr. The south-eastern wind blows at an angle ϕ =60^0 to the equator with velocity v = 15 km/hr. Find the wind velocity v′ relative to the ship and the angle ϕ′ between the equator and the wind direction in the reference frame fixed to the to the ship.

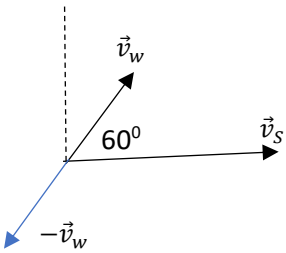
South-eastern wind means the wind blows from south east.

If the frame of reference is fixed with ship we have to find wind velocity relative to the ship. Thus,

$$\vec{v}' = \vec{v}_{WS} = \vec{v}_W - \vec{v}_S$$

Or $\vec{v}' = \vec{v}_W + (-\vec{v}_S)$

(subtracting means adding opposite vector)



Or $v' = \sqrt{v_W^2 + v_s^2 + 2v_w v_s \cos\varphi}$

Or $v' = \sqrt{15^2 + 30^2 + 2 * 15 * 30 * \cos 120^0}$

Or $v' = 26$ km/hr

And the direction will be

And the direction will be given by

$$\tan\varphi' = \frac{v_w \sin 120^0}{v_s + v_w \cos 120^0} = \frac{15*\sqrt{3}}{2*30-15} = \frac{1}{\sqrt{3}}$$

Gives $\varphi' = 30^0$