Q- A wire frame in the shape of an equilateral triangle of side d = 8 cm, carries current i = 0.25 A. This frame is in a region of space that contains a constant magnetic field of magnitude B = 1.3 T, normal to the plane of the frame and the frame is in stable equilibrium. What is the work you would have to do to rotate the frame 180° so that it is again normal to the field?
The problem is based on the phenomenon of the rotation of a dipole in a field.
When a current carrying loop is placed in a uniform magnetic field it may experience a torque. The torque is given by

$$\vec{\tau} = \vec{p}_B \times \vec{B}$$

Magnitude of which is given by

$$\tau = p_B.B.\sin\theta$$

Here p_B is the magnetic dipole moment of the loop, θ is the angle between the field and the dipole and B is the magnetic field. For $\theta = 0$ the torque will be equal to zero and the dipole is in stable equilibrium. The torque on the dipole due to magnetic field acts in such a direction that it tries to bring the dipole in the direction of the field. To keep the dipole making angle θ with the field, we must apply a torque equal to τ but in opposite direction. Now for a small rotation from angle θ to θ + d θ the work to be done by the torque applied will be

$$dW = \tau.d\theta$$

Thus, if the dipole is rotated from an angle θ_1 to θ_2 then the work done is given by

$$W = \int_{\theta 1}^{\theta 2} \tau.d\theta = \int_{\theta 1}^{\theta 2} p_B B\sin\theta.d\theta = -p_B B\left(\cos\theta_2 - \cos\theta_1\right) \text{ -------- (1)}$$

The magnetic dipole moment of a loop is given by the product of its area and current in it hence the dipole moment of the equilateral triangle carrying current I is given by

$$p_B = I * \left(\frac{\sqrt{3}}{4}\right) d^2$$

Hence the work done to rotate the dipole from θ = 0 to $\theta = 180^0$ is given by using equation 1 as

$$W = -p_B B\left(\cos\theta_2 - \cos\theta_1\right)$$

Or $W = -I * \frac{\sqrt{3}}{4}\, d^2 B(\cos 180^0 - \cos 0)$

Or $W = -0.25 * \frac{\sqrt{3}}{4}\,(0.08)^2 * 1.3 * (-1 - 1)$

Or W = $1.8*10^{-3}$ J