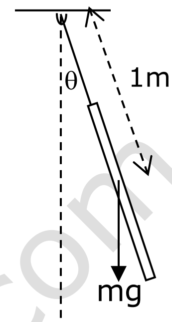
Q- A very light rigid rod with a length of 0.500m extends straight out from one end of a meterstick. The meterstick is suspended from a pivot at the far end of the rod and is set into oscillation in a vertical plane. Determine the period of oscillation.

When a rigid body (not a particle) is suspended from a pivot and free to rotate in a vertical plane, the system is called a physical pendulum or compound pendulum and its time-period T for small oscillation is given by



$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

Here m is the mass of the body and *l* is the distance of the center of mass of the body from the point of suspension. I is the moment of inertia of the body about the axis of rotation passing through the point of suspension.

The distance of the center of mass C (mid-point) of the stick from the point of suspension is

$$l = 0.5 + 0.5 = 1 \text{ m}$$

Moment of inertia of the stick about the point of suspension O is given by the parallel axis theorem which says that the moment of inertia of a body about any axis is equal to the sum of (moment of inertia I_{cm} of the body about a parallel axis passing through its center of mass) and (the product of mass and square of the distance between the two parallel axes md^2).

Thus, $I = I_{cm} + md^2$

Here the axis of rotation is through the point of suspension and is horizontal and normal to the page. The parallel axis from center of mass of the stick will be along the perpendicular bisector of the stick and hence the moment of inertia about this axis will be

$$I_{cm} = \frac{mL^2}{12}$$

Where m is the mass of the stick and L is its length (=1m).
The distance between the two axes is d = *l* = 1 m

Hence the moment of Inertia of the system about the point of suspension, according to parallel axis theorem will be

$$I = I_{cm} + md^2$$

Or $$I = \frac{mL^2}{12} + ml^2 = m\left(\frac{L^2}{12} + l^2\right)$$

Thus, the time period of oscillation of the stick is given by

$$T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{m\left(\frac{L^2}{12} + l^2\right)}{mgl}} = 2\pi\sqrt{\frac{1}{gl}\left(\frac{L^2}{12} + l^2\right)}$$

As L = *l* = 1 m, and g = 9.8 m/s² we have

$$T = 2\pi\sqrt{\frac{1}{9.8}\left(\frac{1}{12} + 1\right)} = 2*3.14\sqrt{\frac{13}{9.8*12}} = 2.09 \text{ s.}$$