Q - A spherical raindrop of radius a, falls from rest under gravity. It falls through a stationary cloud such that because of condensation its radius increases with time at a constant rate k. Find the distance fallen by the raindrop as a function of time t.

Let the radius of the drop at time t be r

The rate of increase of the radius is given by $\frac{dr}{dt} = K$

As the initial radius is a and it increases in time t by Kt the radius r, at time t will be

r = a + K*t

The volume of the drop at time t will be $\frac{4}{3}\pi r^3$

And hence its mass will be m = $\frac{4}{3}\pi(a+Kt)^3 \rho$

Here ρ is the density of water.

Now according to the second law of motion the rate change of momentum is equal to the force applied and the only force acting on the drop is the gravity (neglecting air resistance).

As the mass of the drop is variable, hence taking downward direction positive, we can write

$$F = \frac{dP}{dt} = \frac{d}{dt}(mv) = mg$$

Here v is the velocity of the drop at time t (initial velocity is zero)

Substituting the value of mass from above we get

$$\frac{d}{dt}\left(\frac{4}{3}\pi(a+Kt)^3 \rho * v\right) = \left(\frac{4}{3}\pi(a+Kt)^3 \rho\right) g$$

Or $$\frac{d}{dt}\left\{(a+Kt)^3 * v\right\} = (a+Kt)^3 g$$

Integrating this equation wrt t we get

$$(a+Kt)^3 * v = \frac{g}{4K}(a+Kt)^4 + C$$ Here C is the constant of integration.

As at t = 0; v = 0 substituting in the equation we have $C = -\frac{ga^4}{4K}$

Substituting this in above equation we have

$$(a+Kt)^3 * v = \frac{g}{4K}(a+Kt)^4 - \frac{ga^4}{4K}$$

Or $$(a+Kt)^3 * v = \frac{g}{4K}\left\{(a+Kt)^4 - a^4\right\}$$

Or $$v = \frac{g}{4K}\left\{(a+Kt) - \frac{a^4}{(a+Kt)^3}\right\}$$ ------------------- (2)

Now if the distance fallen in time t is x, then the velocity is given by v = dx/dt and hence

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$$\frac{dx}{dt}=\frac{g}{4K}\left\{(a+Kt)-\frac{a^4}{(a+Kt)^3}\right\}$$

Integrating wrt to t we have

$$x=\frac{g}{4K}\left\{\frac{(a+Kt)^2}{2K}+\frac{a^4}{2K(a+Kt)^2}\right\}+C'$$ Here C′ is constant of integration.

Or $$x=\frac{g}{8K^2}\left\{(a+Kt)^2+\frac{a^4}{(a+Kt)^2}\right\}+C'$$

Now as initially at t = 0 ; x = 0 gives $C'=-\frac{ga^2}{4K^2}$

Substituting in equation above we get

$$x=\frac{g}{8K^2}\left\{(a+Kt)^2+\frac{a^4}{(a+Kt)^2}\right\}-\frac{ga^2}{4K^2}$$

Or $$x=\frac{g}{8K^2}\left\{(a+Kt)^2+\frac{a^4}{(a+Kt)^2}-2a^2\right\}$$

Or $$x=\frac{g}{8K^2}\left\{(a+Kt)-\frac{a^2}{(a+Kt)}\right\}^2$$

Or $$x=\frac{g}{8K^2}\left\{\frac{(a+Kt)^2-a^2}{(a+Kt)}\right\}^2$$

Or $$x=\frac{gt^2}{8}\left\{\frac{2a+Kt}{a+Kt}\right\}^2$$

This is the required distance.

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